Operationalisation TSA for use in discrete spectral wave models

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North Shore, 11-16 Nov. 2007

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Motivation

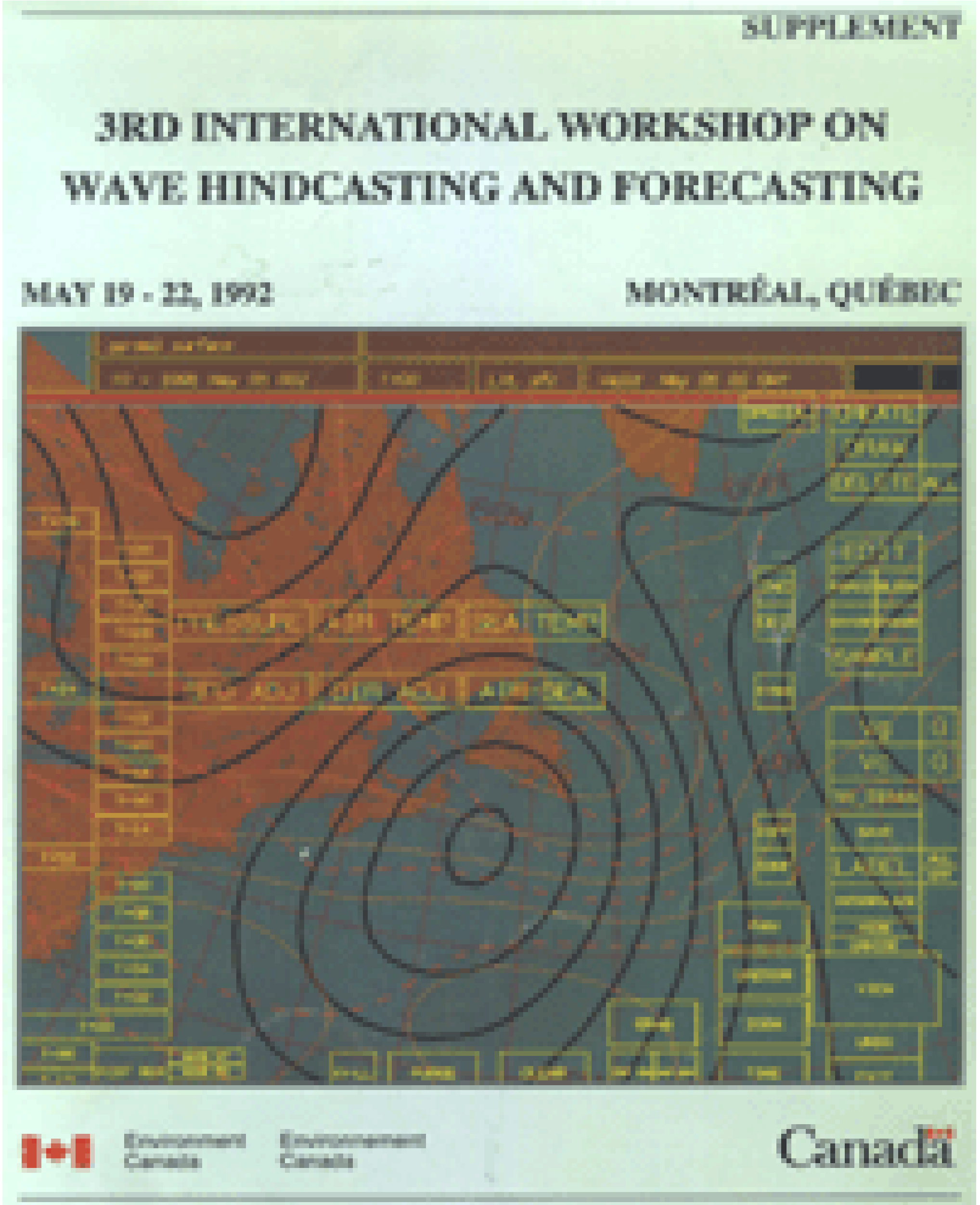
MORPHOS: Improve modeling of physical processes in wave models

Focus on S_{nl4}

DIA is fast but only crude approximation
Xnl accurate but very time consuming

DIA in spectral models hamper further developments

Replace the DIA by a more accurate and computationally fast method: TSA method good candidate (Resio & Perrie 2006)


SUPPLEMENT
3RD INTERNATIONAL WORKSHOP ON
WAVE HINDCASTING AND FORECASTING
MAY 19 - 22, 1992
MONTRÉAL, QUÉBEC
Environment Canada
Environnement Canada
Canada

15 years ago

1992

A GENERIC THIRD-GENERATION WAVE MODEL: AL

D. Resio,[1] W. Perrie,[2] S. Thurston,[1] and J. Hubertz[3]

[1]Florida Institute of Technology
Melbourne, Florida

[2]Bedford Institute of Oceanography
Bedford, Nova Scotia

[3]U.S. Army Engr. Coastal Engineering Research Center
Vicksburg, Mississippi

Two scales

Let us consider a spectrum which is represented as the sum of two terms at each point within the spectrum, i.e.

$$9)\quad n(\underline{k}) = \overline{n(\underline{k})} + n'(\underline{k}) \qquad (9)$$

where the overbar denotes a broad-scale averaging and the prime denotes a local departure from the broad-scale structure. The action density term in equation 3 can now be represented in an expanded form as

$$
\begin{aligned}
&10)\, D(\underline{k}_1, \underline{k}_2, \underline{k}_3, \underline{k}_4) = \qquad (10)\\
&\overline{n(\underline{k}_1)\, n(\underline{k}_3)}\, [\overline{n(\underline{k}_4)} - \overline{n(\underline{k}_2)}] + \overline{n(\underline{k}_2)\, n(\underline{k}_4)}\, [\overline{n(\underline{k}_3)} - \overline{n(\underline{k}_1)}]\\
&[\overline{n(\underline{k}_1)}\, n'(\underline{k}_3) + n'(\underline{k}_1)\, \overline{n(\underline{k}_3)}]\ [\overline{n(\underline{k}_4)} - \overline{n(\underline{k}_2)} + n'(\underline{k}_4) - n'(k_2)]\\
&+ \overline{n(\underline{k}_1)\, n(\underline{k}_3)}\, (n'(\underline{k}_4) - n'(\underline{k}_2)] + n'(\underline{k}_1)\, n'(\underline{k}_3)\, [\overline{n(\underline{k}_4)} - \overline{n(\underline{k}_2)}]\\
&+ \overline{n(\underline{k}_2)\, n(\underline{k}_2)}\, [n'(\underline{k}_3) - n(\underline{k}_1)] + n'(\underline{k}_2)\, n'(\underline{k}_4)\, [\overline{n(\underline{k}_3)} - \overline{n(\underline{k}_1)}]\\
&[\overline{n(\underline{k}_2)}\, n'(\underline{k}_4) + n'(\underline{k}_2)\, \overline{n(\underline{k}_4)}]\ [\overline{n(\underline{k}_3)} - \overline{n(\underline{k}_1)} + n'(\underline{k}_3) - n'(\underline{k}_1)]\\
&+ n'(\underline{k}_1)\, n'(\underline{k}_3)\, [n'(\underline{k}_4)\, n'(\underline{k}_2)] + n'(\underline{k}_2)\, n'(\underline{k}_4)\, [n'(\underline{k}_3) - n'(\underline{k}_1)]
\end{aligned}
$$

as before the overbar here denotes a broad-scale feature of the spectrum and

Purpose

Operationalisation of TSA method for the computation of non-linear four-wave interactions in operational discrete spectral third-generation wave models (WaveWatch, SWAN, STWAVE, WAM, ...)

Challenge

Turn a research code into a flexible operational code in the form of a subroutine for general use

Principle of the TSA

- Split an arbitrary spectrum into two part: a broad-band spectrum and a residual spectrum;
- Compute non-linear transfer rate using pre-computed exact transfer rates and correction terms;
- The non-linear transfer rate of the broad-band spectrum are pre-computed (first scale) and stored in a database;
- Computation of correction terms based on product terms of spectral densities of the broad-band spectrum, residual spectrum and pre-computed terms (second scale).

Description of the TSA

Split spectrum into broad-band and perturbation

$$n_i = b_i + p_i \quad \text{for} \quad i = 1,4$$

Mathematical structure of TSA derived from WRT method

$$\frac{\partial n_1}{\partial t} = B(\boldsymbol{k}_1) + \iint (p_3 - p_1)\Lambda_d(\boldsymbol{k}_1, \boldsymbol{k}_3) k_3 dk_3 d\theta_3$$

$$+ \iint (p_1 p_3 + p_1 b_3 + b_1 p_3)\Lambda_p(\boldsymbol{k}_1, \boldsymbol{k}_3) k_3 dk_3 d\theta_3$$

$$+ \quad \ldots\ldots$$

Requirements of TSA subroutine

- Applicable in any discrete spectral wave model
- Input arrays: flexible range of frequencies, directions, depth, power of tail
- Output: non-linear transfer rate
- Settings: internal loops, fitting of spectra, location of database, logging options
- Error messaging
- Fortran 95, dynamic memory allocation, modules
- Documentation

Discretisation and filtering

$$\Delta n\left(k_{i_{k1}},\theta_{j_{k1}}\right) = B\left(k_{i_{k1}},\theta_{j_{k1}}\right)$$

$$+\sum_{i_{k3}=1}^{N_k}\sum_{j_{k3}}^{N_\theta}\left(p_{3i_{k3},j_{k3}} - p_{1i_{k3},j_{k3}}\right)\Lambda_a\left(i_{k1}, j_{k1}, i_{k3}, j_{k3}\right)k_{i_{k3}}\Delta k_{i_{k3}}\Delta\theta$$

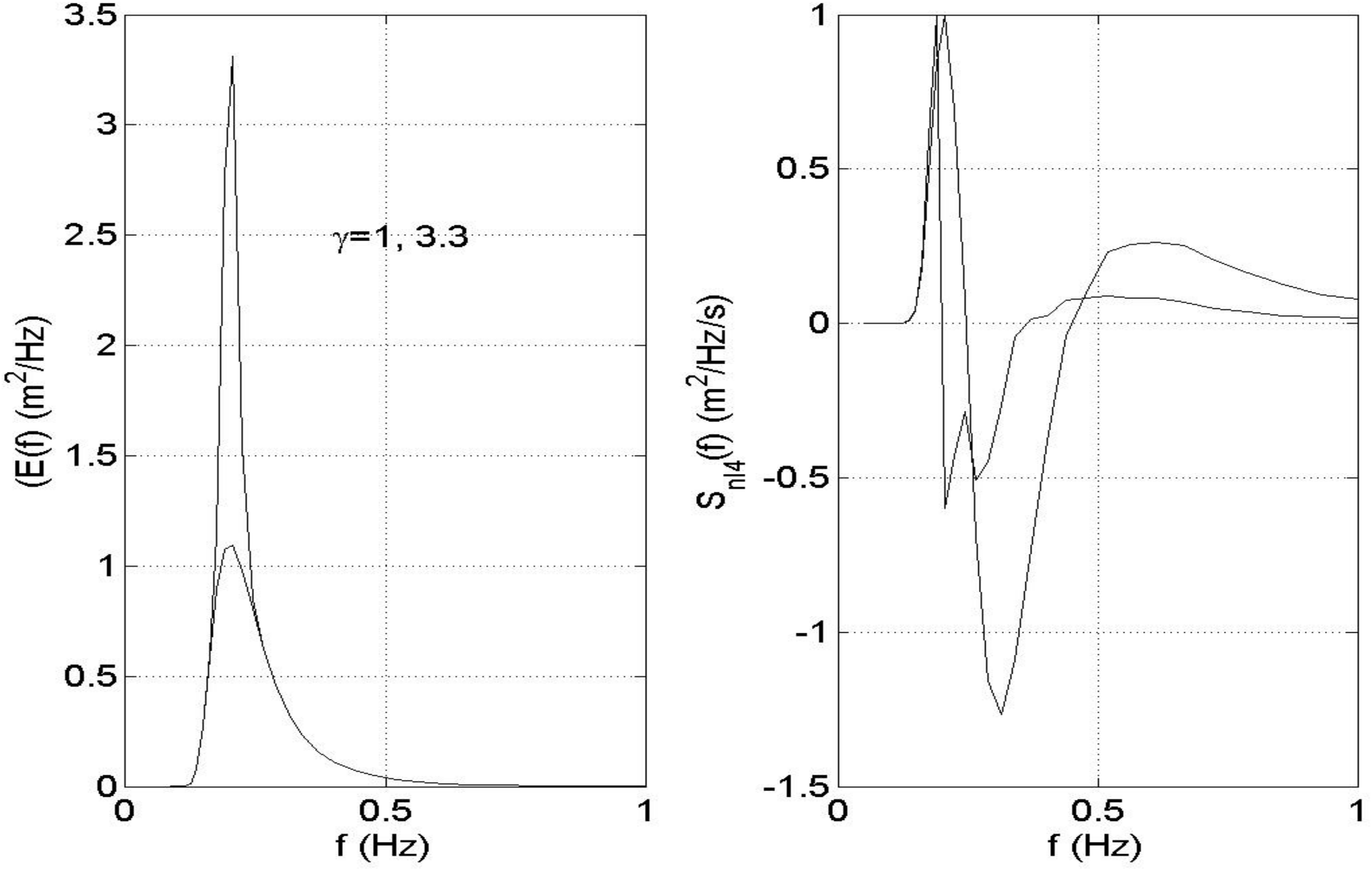
$$+\sum_{i_{k3}=1}^{N_k}\sum_{j_{k3}}^{N_\theta}\left(p_{1i_{k1},j_{k1}}p_{3i_{k3},j_{k3}} + b_{1i_{k1},j_{k1}}p_{3i_{k3},j_{k3}} + p_{1i_{k1},j_{k1}}b_{3i_{k3},j_{k3}}\right)\Lambda_b\left(i_{k1}, j_{k1}, i_{k3}, j_{k3}\right)k_{i_{k3}}\Delta k_{i_{k3}}\Delta\theta$$

Omit contributions when $\boldsymbol{k}_1$ and $\boldsymbol{k}_3$ are well separated in wave number space

$$k_* = \frac{k_1 - k_3}{k_p} < k_F$$

$$\theta_* = \left[\theta_1 - \theta_2\right] < \theta_F$$

Non-linear transfer rate depends on peakedness but also on directional spreading



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Scaling laws

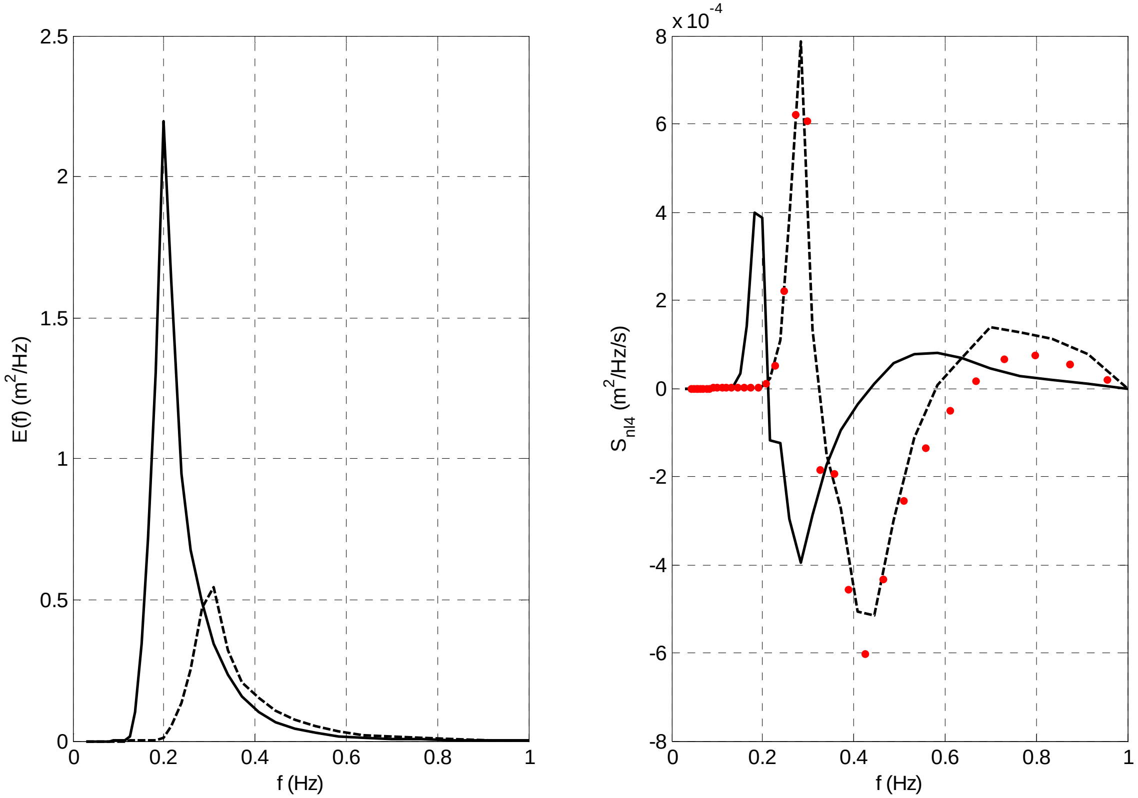
Non-linear transfer rate of similar spectra are related via scaling laws and rotations.

$$E\left(f,\theta\right)=\alpha f_p^{-n}\Psi\left(\upsilon,\theta\right)$$

$$S_{nl}\left(f,\theta\right)=\alpha^3 f_p^{11-3n}\Omega\left(\upsilon,\theta\right)$$

$$S_{nl}^{(2)}\left(f,\theta\right)=S_{nl}^{(1)}\left(f\frac{f_{p1}}{f_{p2}},\theta-\Delta\theta\right)\left(\frac{\alpha_2}{\alpha_1}\right)^3\left(\frac{f_{p2}}{f_{p1}}\right)^{-4}$$

Scaling of non-linear transfer rate for spectra with different α and f_p



Red dots are transformed transfer rates

Small mismatch at higher frequencies

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Elements of TSA subroutine

- Determine characteristics of arbitrary spectrum: f_p, α, peakedness γ, mean direction θ and directional spreading σ

- search best fitting database in terms of peakedness, mean direction and directional spreading

- split spectrum into broad-band (available in database) and residual part

- retrieve non-linear transfer rate and related matrices

- apply scaling laws and directional transformation

- compute correction terms to broad-band transfer rate

- add broad-band transfer rate and correction terms

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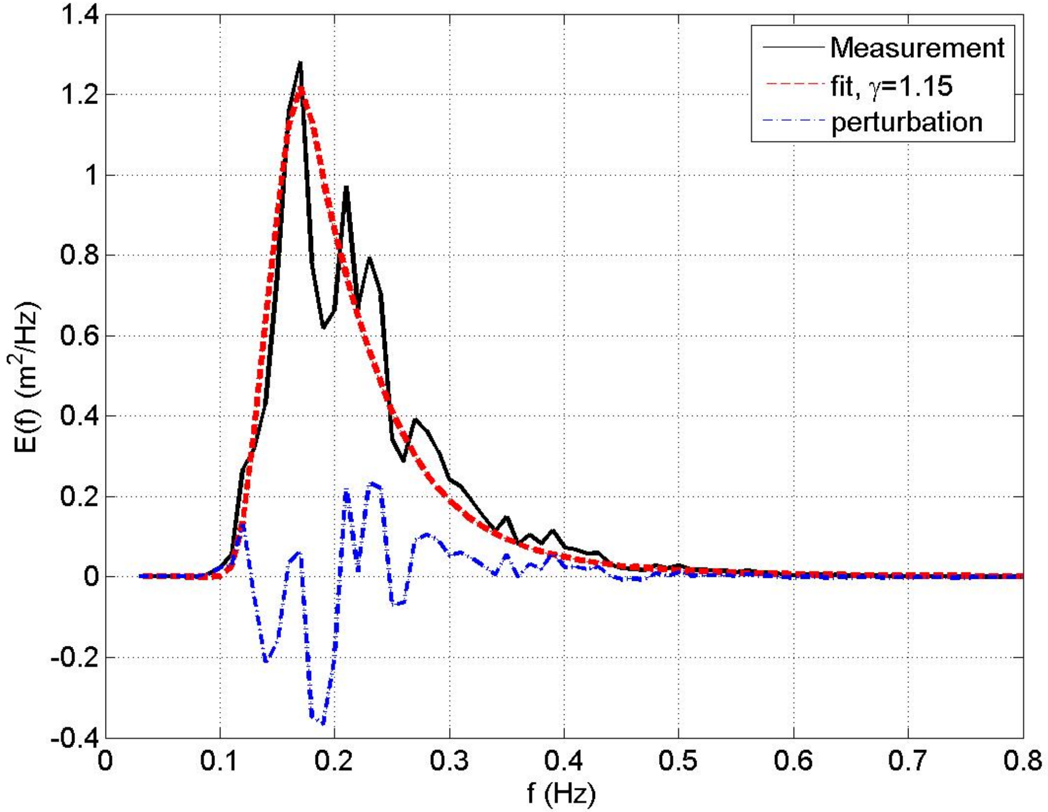
Fitting of broad-band spectrum to an arbitrary spectrum



Fitted γ (1.15) generally not in database, but only 'round' values, e.g. 1, 1.25, 1.5, 1.75, 2.0, etc.

Use γ=1.25 for broad band spectrum, rescale to conserve energy and re-computed residual spectrum

Similar arguments hold for directional spreading and mean direction ($\Delta\theta$)

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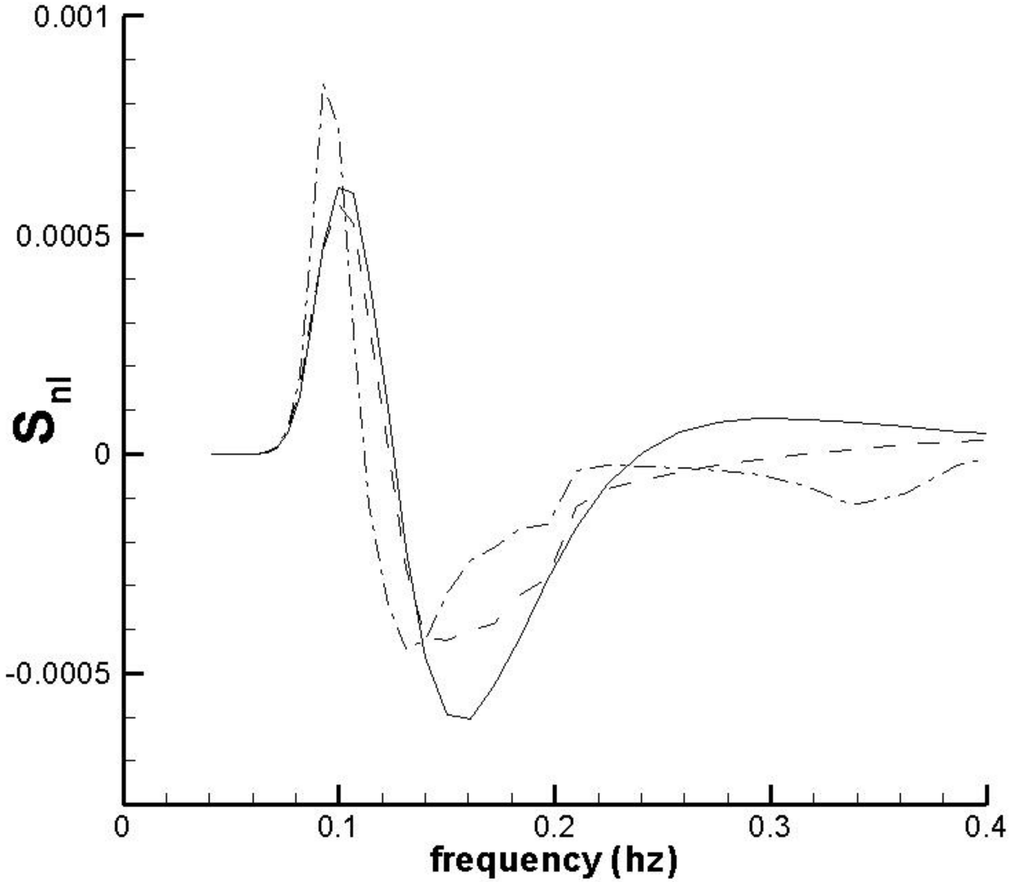
Comparison TSA versus Xnl (Resio & Perrie, 2006)



___ exact
-.-.-. broad-band
- - - TSA

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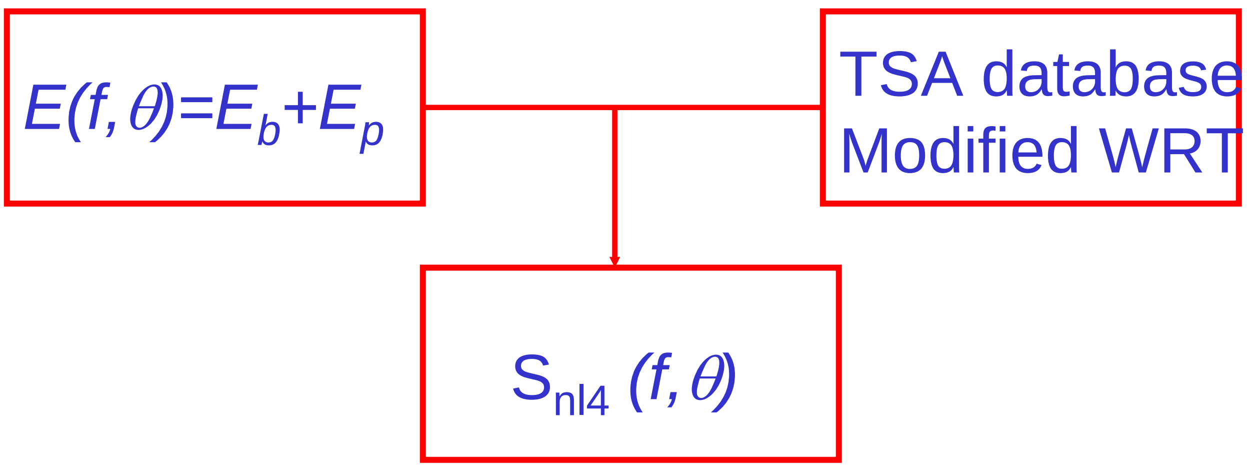
Set-up of TSA for a specific model application

- Choose frequencies f_i and directional step $\Delta\theta$;
- Choose range of peakedness values γ, and directional spreading σ to set-up database;
- Use modified WRT method to compute exact nonlinear transfer rates and related matrices Λ_d and Λ_p;
- Choose settings for internal loops (related to filtering out of negligible contributions);
- Specify location of pre-computed database.

Computational procedure

1: choose spectral grid (typically 30 f and 36 θ)
2: compute broadband transfer rates and TSA matrices
3: compute transfer rate for given spectrum



Physical aspects of TSA implementation

- How many databases need to be pre-computed, related to the question: how far may the arbitrary spectrum deviate from the broad-band spectrum ?
- Optimal range of inner loops, .e.g. filtering of negligible contributions
- Inclusion of shallow water effects, not needed for core of TSA, but only related to extent of parameters of database, *kh* values.

Outlook

- Testing fitting procedure to obtain broad band spectrum
- Testing scaling and transformation of spectra and transfer rates
- Optimization of file i/o (keep data in memory when possible)
- Implementation in operational models (SWAN, WaveWatch, STWAVE, WAM)

- Growth curve analysis, good performance for an individual spectrum is no guarantee that it works in a model run
- Field cases with complex situations: turning winds, slanting fetch (Duck data), hurricanes (Katrina, Rita)
- Assessment of model improvement (parameters, spectral shape)
- Available spring 2008