Extension of ECMWF freak wave warning system to 2 dimensions

Peter Janssen, Nobuhito Mori, and Miguel Onorato

European Centre for Medium-Range Weather Forecasts

<peter.janssen@ecmwf.int>

INTRODUCTION

In october 2003 ECMWF introduced the first operational freak wave warning system, which was based on a parametrization of the one-dimensional version of the Benjamin-Feir Instability. Therefore, parameters such as maximum wave height were, apart from the number of waves in the timeseries, only dependent on the **Benjamin-Feir Index** (BFI), which is basically the ratio of wave steepness to the width of the frequency spectrum. But effects of directional width are important as well.

The programme of this talk is as follows:

- **BRIEF REVIEW of THEORY**
- **SHORT and LARGE TIME KURTOSIS EVOLUTION** Application of narrow-band version of theory for fixed spectral shape
- **MONTE CARLO SIMULATIONS of NLS** Present Monte Carlo Simulations and derive a simple parametrization for kurtosis as function of BFI and directional width.

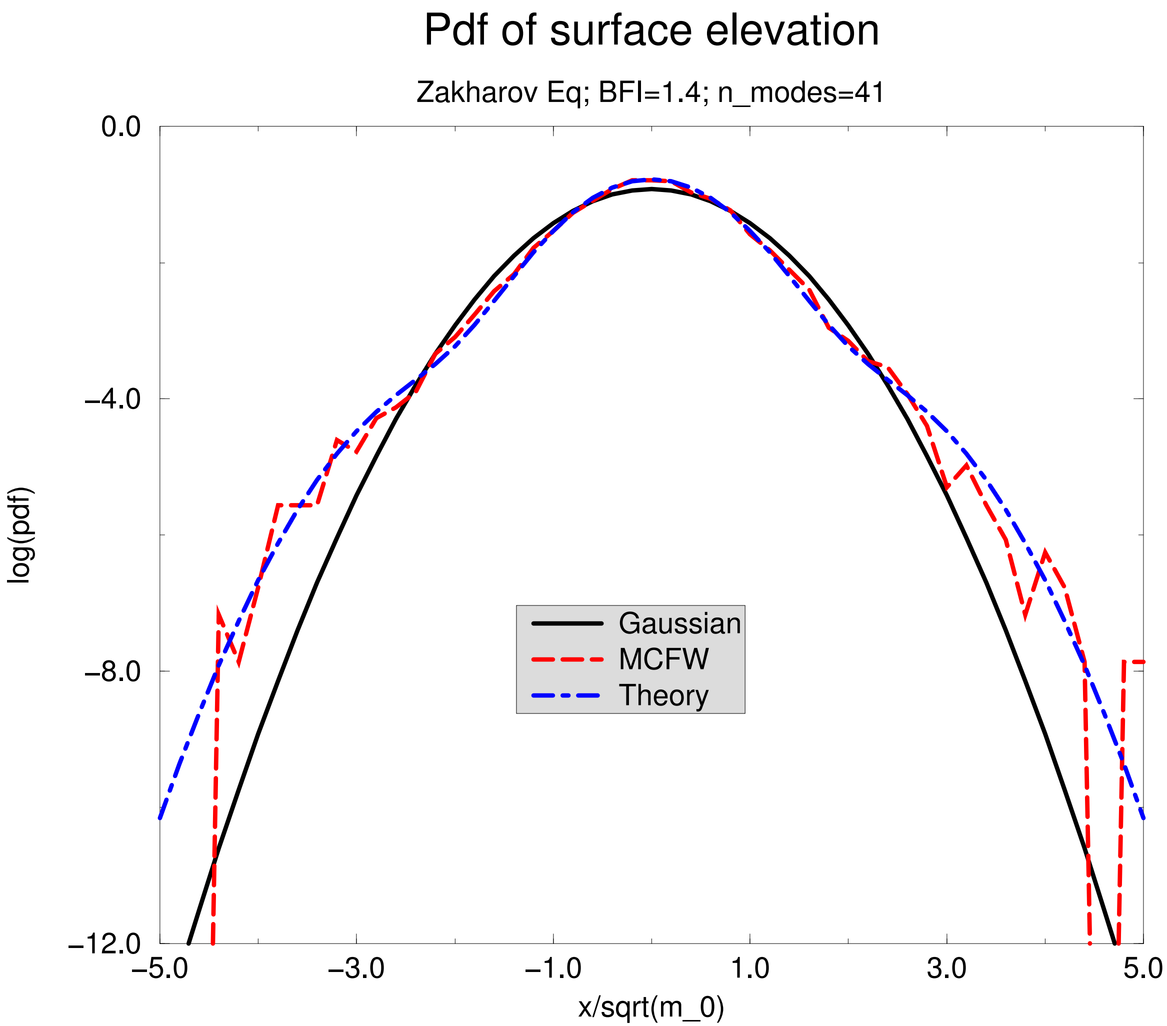
- **MAXIMUM WAVE HEIGHT** Obtain from the pdf of the envelope the expection value of **maximum wave height** and corresponding **maximum wave period**.
- **RESULTS and FIRST VALIDATION** Validate average results against North Sea Buoy data.
- **CONCLUSIONS**

THEORY of NONLINEAR FOCUSSING

- Freak waves are examples of extreme, nonlinear ocean waves which may cause considerable damage to large vessels.
- These extreme waves are generated by *nonlinear focussing*, a process that also causes the Benjamin-Feir Instability.

Linear waves on the open ocean are independent and therefore the **Random Phase Approximation** applies. This means that in a good approximation ocean waves follow Gaussian statistics.

However, nonlinear interactions $\rightarrow$ correlated phases $\rightarrow$ deviations from Gaussian sea state. The kurtosis (which measures deviations from the Normal distribution) can be expressed in terms of the wave spectrum, and therefore for given sea state the probability of occurrence of extreme events can be obtained. Theory has been validated against Monte Carlo Simulations using the Zakharov Equation.



Log of PDF for surface elevation (BFI=1.4). For reference the Gaussian distribution is shown as well. Freak waves correspond to a normalized height of 4 or larger.

Zakharov Equation

Start from a Hamiltonian description of ocean waves and write the surface elevation η in terms of the action variable $A(\vec{k}, t)$,

$$\eta = \int_{-\infty}^{\infty} \mathrm{d}\vec{k} \left(\frac{k}{2\omega} \right)^{1/2} [A(\vec{k}) + A^*(-\vec{k})] e^{i\vec{k}.\vec{x}},$$

with $\vec{k}$ the wave number and $\omega = \sqrt{gk}$. Apply Krasitskii's transformation, $A = A(a, a^*)$, to remove the nonresonant, bound-wave contributions and then Hamilton's equations $i\partial a/\partial t = \delta E/\delta a^*$ become the well-known Zakharov equations

$$\boxed{\frac{\partial}{\partial t} a_1 + i\omega_1 a_1 = -i \int \mathrm{d}\vec{k}_{2,3,4} T_{1,2,3,4} a_2^* a_3 a_4 \delta_{1+2-3-4}.}$$

Note: time scale is $\mathcal{O}(1/\epsilon^2)$!!,where ϵ is the wave steepness; Four-wave interactions!

Stochastic Approach

In wave forecasting we are interested in predicting quantities such as the second moment

$$B_{1,2} =< a_1 a_2^* >,$$

where angle brackets denote an ensemble average. Following methods employed in Statistical Mechanics (Liouville $\rightarrow$ Boltzmann) one obtains from the deterministic Zakharov equation an equation for the action density N, where for a homogeneous sea

$$B_{1,2} = N_1 \delta(\vec{k}_1 - \vec{k}_2).$$

The equation for action density $N(\vec{x}, \vec{k}, t)$becomes:

$$\frac{\partial}{\partial t}N_4 = 4\int \mathrm{d}\vec{k}_{1,2,3}T^2_{1,2,3,4}\delta(\vec{k}_1+\vec{k}_2-\vec{k}_3-\vec{k}_4)\mathcal{R}_i(\Delta\omega,t)$$
$$\times\left[N_1N_2(N_3+N_4)-N_3N_4(N_1+N_2)\right],$$

where $\Delta\omega = \omega_1+\omega_2-\omega_3-\omega_4$. This evolution equation is usually called the Boltzmann equation.

Note there are now two timescales implied by

$$\mathcal{R}_i(\Delta\omega,t) = \sin(\Delta\omega t)/\Delta\omega$$

- short times: $\lim_{t\to 0}\mathcal{R}_i(\Delta\omega,t) = t$, hence $T_{NL} = O(1/\epsilon^2\omega_0)$, the Benjamin-Feir timescale, corresponding to **non-resonant** interactions.
- large times: $\lim_{t\to\infty}\mathcal{R}_i(\Delta\omega,t) = \pi\delta(\Delta\omega)$, corresponding to **resonant** wave-wave interactions, hence $T_{NL} = O(1/\epsilon^4\omega_0)$ (Hasselmann, 1962).

Statistics

Nonlinear transfer gives rise to deviations from Normality which are most conveniently expressed by means of the kurtosis

$$C_4 = <\eta^4> / 3 <\eta^2>^2 - 1,$$

The contribution to C_4 because of dynamics becomes

$$\boxed{C_4^{dyn} = \frac{4}{g^2 m_0^2} \int \mathrm{d}\vec{k}_{1,2,3,4} T_{1,2,3,4} \delta_{1+2-3-4} \left(\omega_1 \omega_2 \omega_3 \omega_4\right)^{\frac{1}{2}} \times \mathcal{R}_r(\Delta\omega, t) N_1 N_2 N_3,}$$

where

$$\mathcal{R}_r(\Delta\omega, t) = \frac{1 - \cos(\Delta\omega\ t)}{\Delta\omega}.$$

The kurtosis is determined by both resonant and non-resonant interactions! However, evaluation of the present form for the kurtosis is far too involved (it is more complicated than the nonlinear transfer). So for operational evaluation approximations are required.

Approximate kurtosis

As a first step we considered the case of one-dimensional propagation. Then, for Gaussian-shaped spectra in the narrow band approximation the kurtosis shows a particularly simple form:

$$\boxed{C_4 = \frac{\pi}{3\sqrt{3}} \times BFI^2,}$$

hence the kurtosis depends on the square of the BF index. Here,

$$BFI = \frac{\epsilon\sqrt{2}}{\delta_\omega}.$$

where the steepness $\epsilon = k_0\sqrt{m_0}$ and $\delta_\omega = \sigma_\omega/\omega_p$ is the relative width of the frequency spectrum.

The one-dimensional case, including validation against lab observations, is discussed by Mori at this meeting.

Next, consider extension to two dimensions.

Extension to 2D

Introduce frequency spectrum $E(\omega)[F = \omega N/g, E = kF/v_g]$ and include effects of bound waves. Using the general result the kurtosis in the narrow band approximation becomes:

$$C_4 = C_4^{dyn} + 8\epsilon^2.$$

with

$$C_4^{dyn} = 4\epsilon^2\omega_0 \int \mathrm{d}\nu_1 \mathrm{d}\nu_2 \mathrm{d}\nu_3 \mathrm{d}\phi_1 \mathrm{d}\phi_2 \mathrm{d}\phi_3 \, \mathcal{R}_r(\Delta\omega, t) \, \hat{E}_1 \hat{E}_2 \hat{E}_3.$$

with $\hat{E} = E(\nu, \phi)/m_0$ the the normalized frequency direction spectrum, $\nu = (\omega - \omega_p)/(\omega_p\delta_\omega)$ and $\phi = (\theta - \theta_p)/\delta_\theta$. Here, δ_ω and δ_θ are the width of the spectrum in the frequency direction and propagation direction respectively. In order to measure the importance of the directional width we introduce the parameter

$$R = \frac{1}{2}\frac{\delta_\theta^2}{\delta_\omega^2}$$

and the frequency mismatch $\Delta\omega$ becomes:

$$\Delta\omega = \delta_\omega^2 \omega_0 \left\{(\nu_3 - \nu_1)(\nu_3 - \nu_2) - R(\phi_3 - \phi_1)(\phi_3 - \phi_2)\right\} + \mathcal{O}(\delta^3),$$

Finally, the resonance function $\mathcal{R}_r$ reads:

$$\mathcal{R}_r(\Delta\omega, t) = \frac{1 - \cos(\Delta\omega\, t)}{\Delta\omega}. \tag{1}$$

The “general” case is very time consuming to solve, consider only special cases of short times and large times.

Short and large times

For short times one finds the general result

$$\boxed{C_4 = \tau^2 BFI^2 \,(1 - R)\,.}$$

where τ is a dimensionless time.

Hence, when $R < 1$ $(\delta_\theta < \sqrt{2}\delta_\omega)$ we have focussing and positive kurtosis and **freak wave formation** while for $R > 1$ we have negative kurtosis.

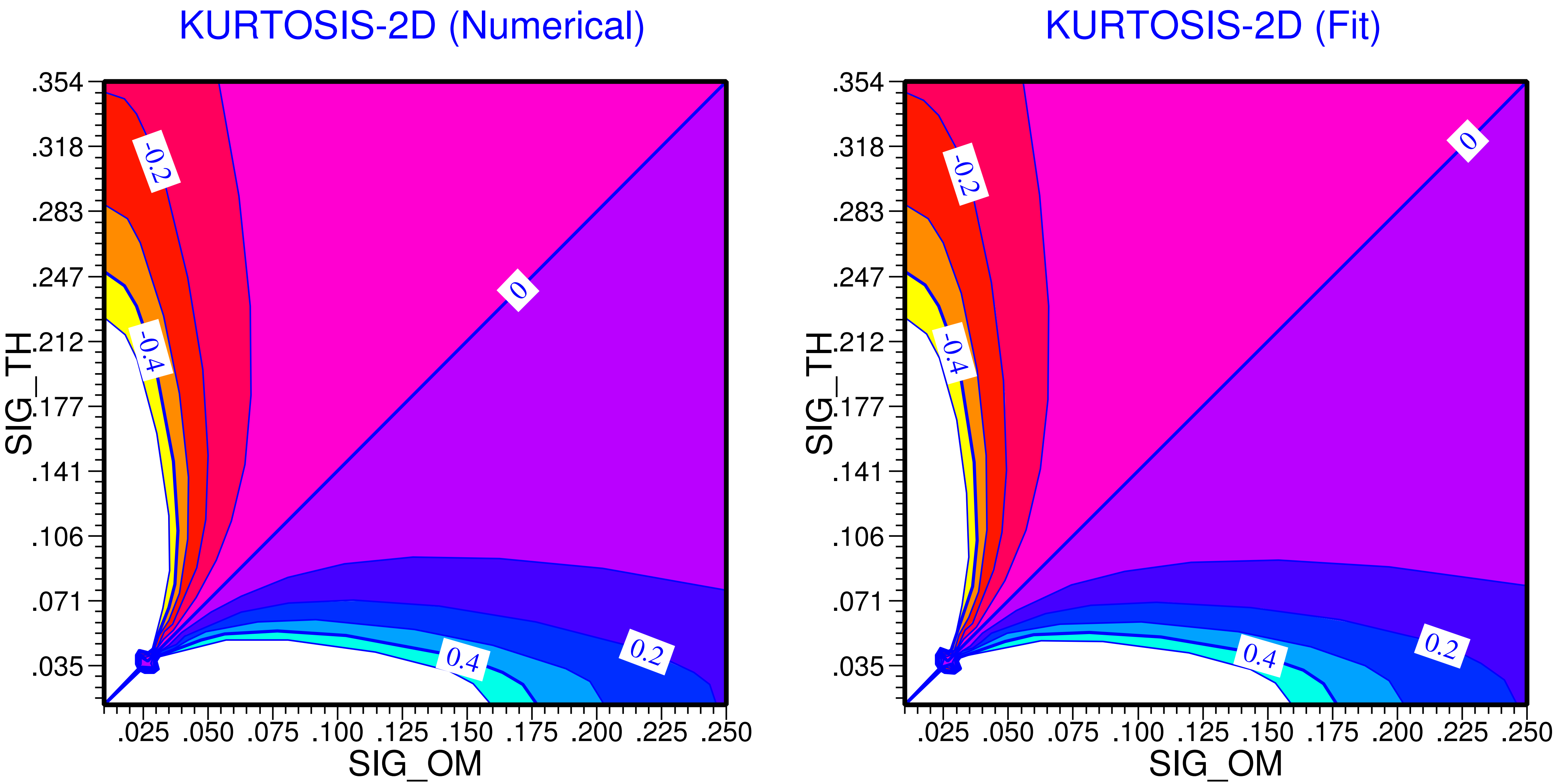
The large time limit is much more involved. Assuming that during that time the spectrum has a Gaussian shape and does not change one finds:

$$C_4 = J(R)\ BFI^2,$$

where

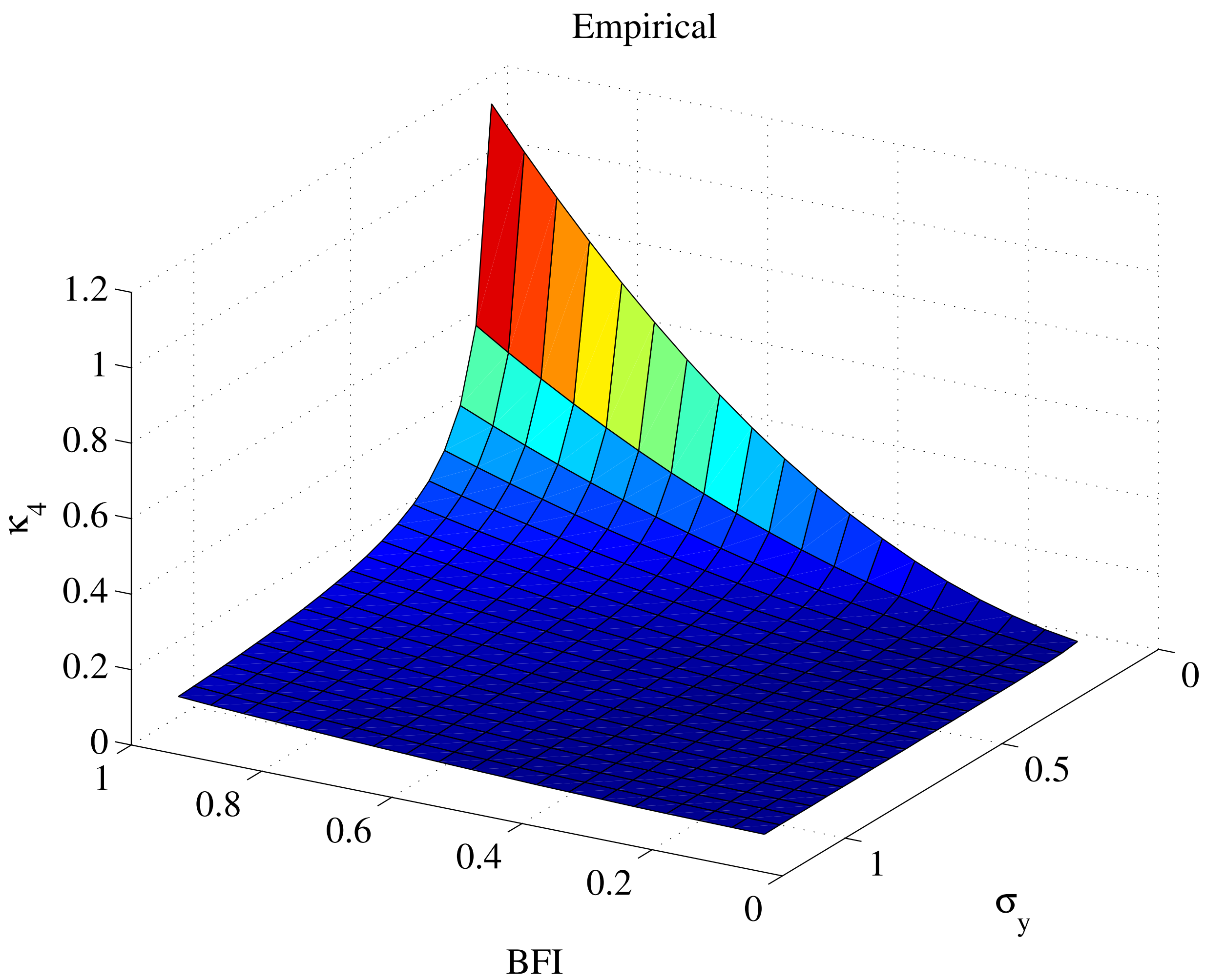
$$J(R) = \frac{1}{(2\pi)^2}\frac{1-R}{R+R_0},$$

with $R_0 = 3\sqrt{3}/4\pi^3$.


KURTOSIS-2D (Numerical)
KURTOSIS-2D (Fit)
SIG_TH
SIG_OM
.354
.318
.283
.247
.212
.177
.141
.106
.071
.035
.025
.050
.075
.100
.125
.150
.175
.200
.225
.250
-0.2
-0.4
0
0.4
0.2

Nonlinear Schrödinger simulations

However, the assumption that the spectrum does not change in time is not correct as was found out by doing $\pm 20,000$ simulations with the Nonlinear Schrödinger Equation. In particular, when $R > 1$ hence frequency width smaller than directional width, there are due to the Benjamin-Feir Instability rapid changes (broadening in the frequency direction) such that **kurtosis flips from negative to positive**. Next viewgraph shows a plot of maximum of Kurtosis as function of BFI and δ_θ suggesting that the maximum is always positive.


Empirical
1.2
1
0.8
0.6
0.4
0.2
0
κ_4
1
0.8
0.6
0.4
0.2
0
BFI
0
0.5
1
σ_y

OPERATIONAL IMPLEMENTATION AND VERIFICATION

Dependence of kurtosis on BFI and directional width was parametrized and implemented in a test version of the ECMWF wave forecasting system.

We use at the moment the following parametrization:

$$C_4 = \frac{0.031}{\delta_\theta} \times \frac{\pi}{3\sqrt{3}} BFI^2,$$

therefore, finite directional width δ_θ is seen to give a considerable **reduction** in kurtosis C_4.

In addition, following Janssen and Onorato (2007), **shallow water** effects are included as well.

Maximum Waveheight

In addition, software was developed to determine maximum wave height and wave period for given pdf of the surface elevation, which includes kurtosis effects.

Proceed as follows (see Mori and Janssen, 2006):

- Start from the pdf of surface elevation, which is the **Gram-Charlier** expansion, i.e. pdf depends on skewness and kurtosis.
- Obtain the pdf of 'wave height' defined as twice the envelope:

$$p(H) = 4H \exp(-2H^2) \left[1 + C_4 A_H(H)\right]$$

where

$$A_H(H) = 2H^4 - 4H^2 + 1$$

Note that because of symmetries the pdf of H does not contain skewness.

- Maximum wave height distribution follows from

$$p_m(H_{max}) = N\left[1 - P(H_{max})\right]^{N-1} p(H_{max})$$

where $P(H) = \int_H^\infty \mathrm{d}h\ p(h)$ is the exceedence probability of wave height, and N is the number of waves. In the continuum limit this becomes

$$p_m(H_{max}) = Np(H_{max}) \times \exp\left[-NP(H_{max})\right]$$

with $B_H(H) = 2H^2\left(H^2 - 1\right)$

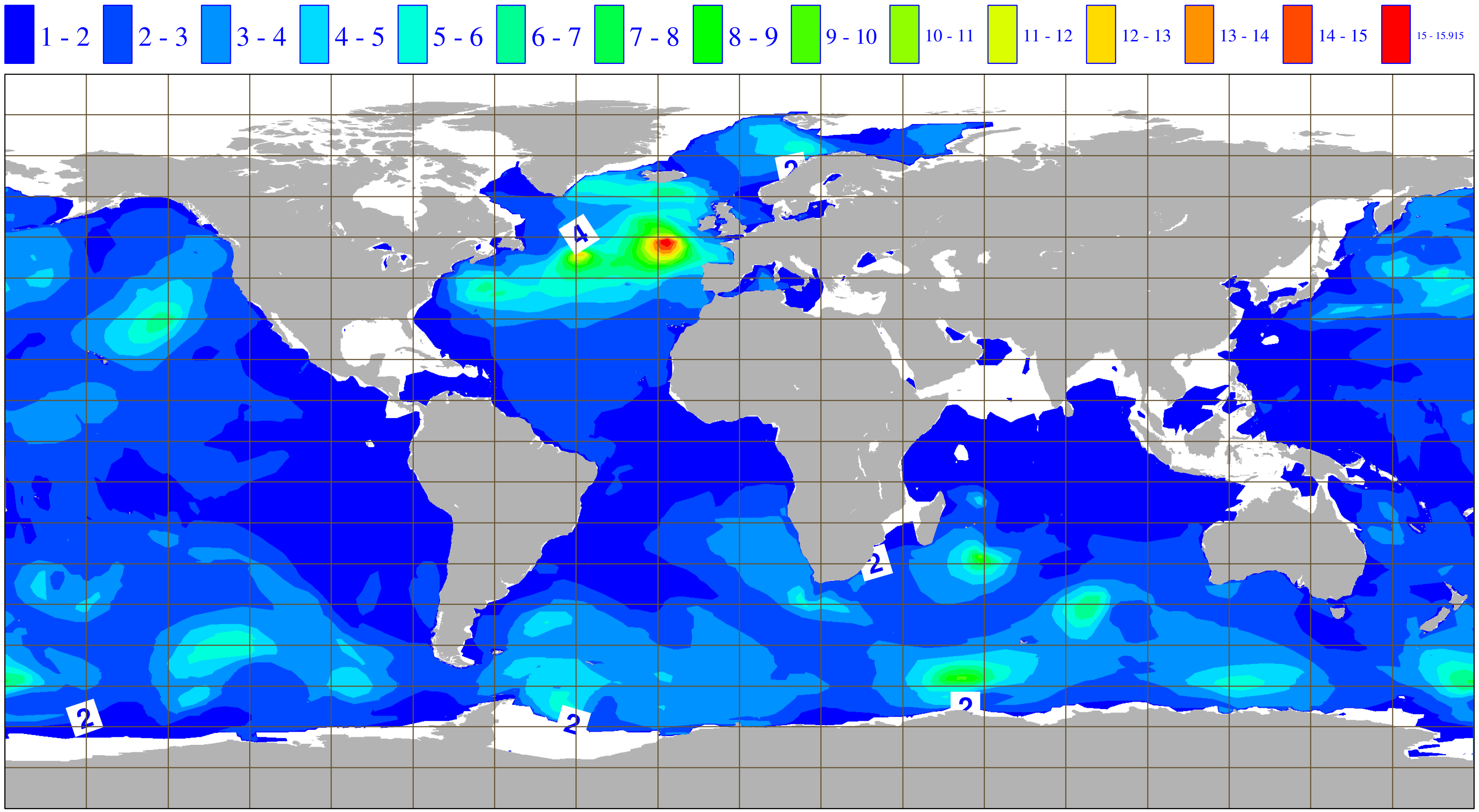
- Expectation value of maximum wave height follows from

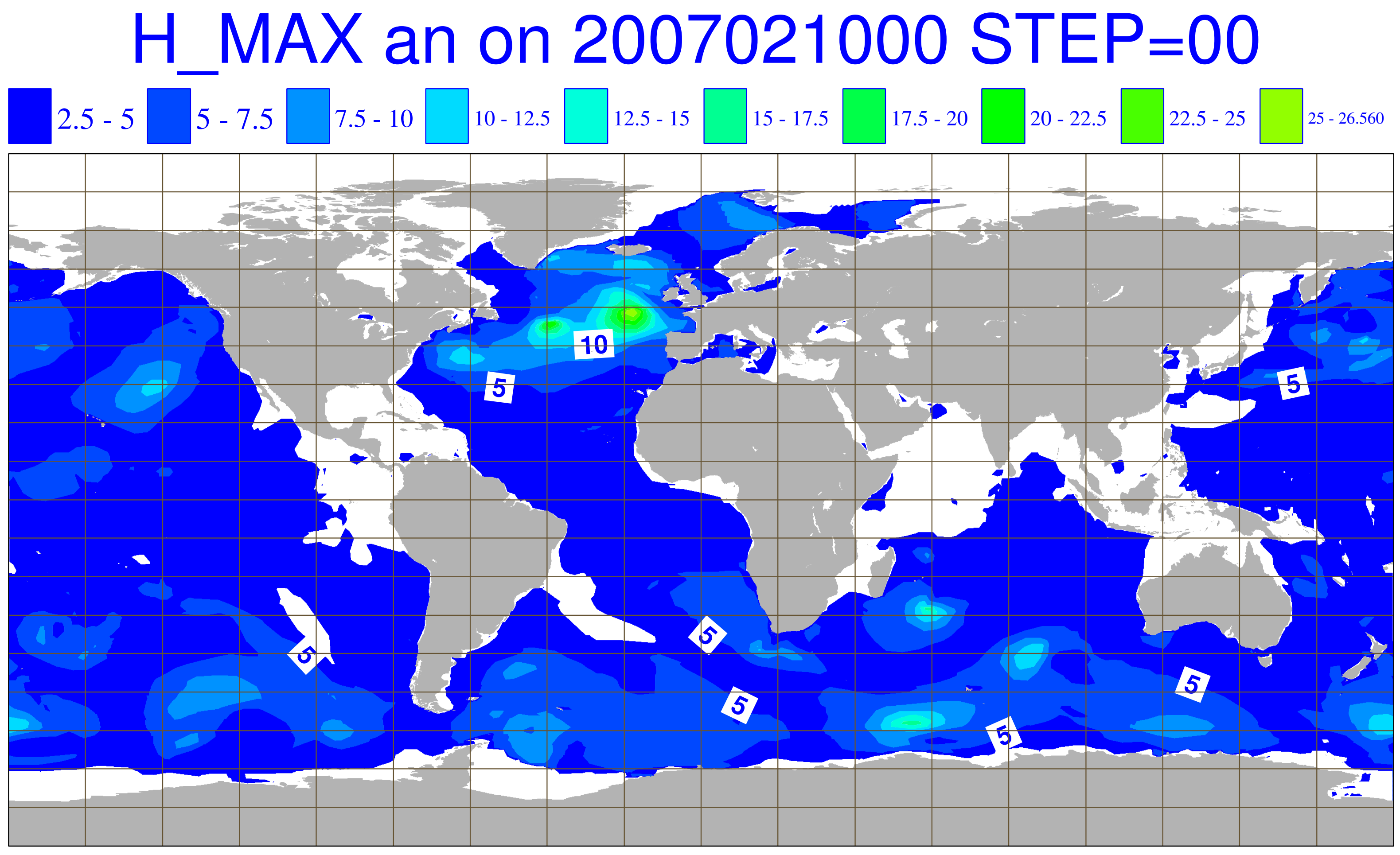
$$\langle H_{max}\rangle = \int_0^\infty \mathrm{d}H_{max}\ H_{max}\ p_m(H_{max})$$

Note: $H_{max} = F[C_4(BFI, R), N]$, where $N = 1/T_p$ with T_p the peak period

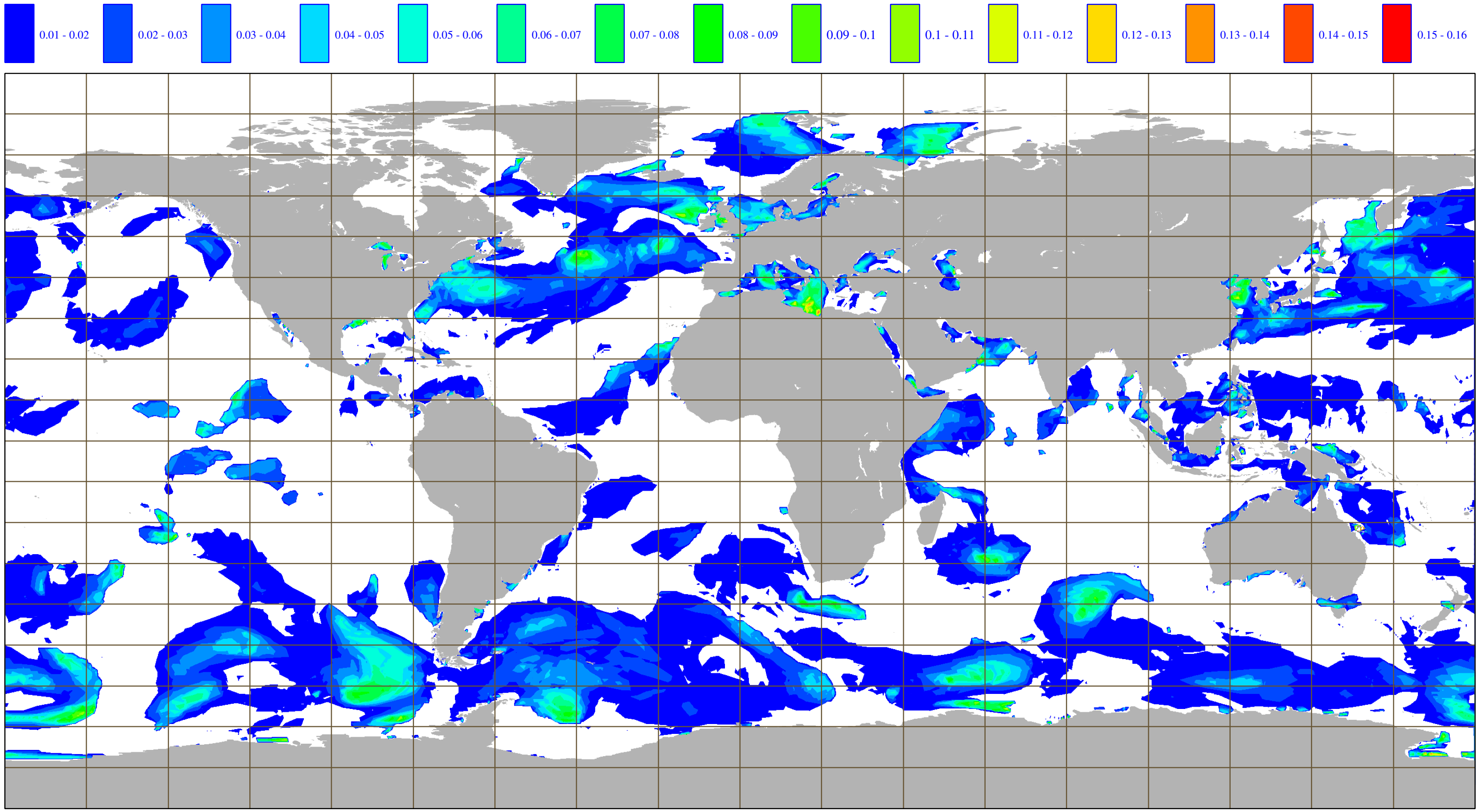
Results of this system for one synoptic time are displayed in the next viewgraphs.

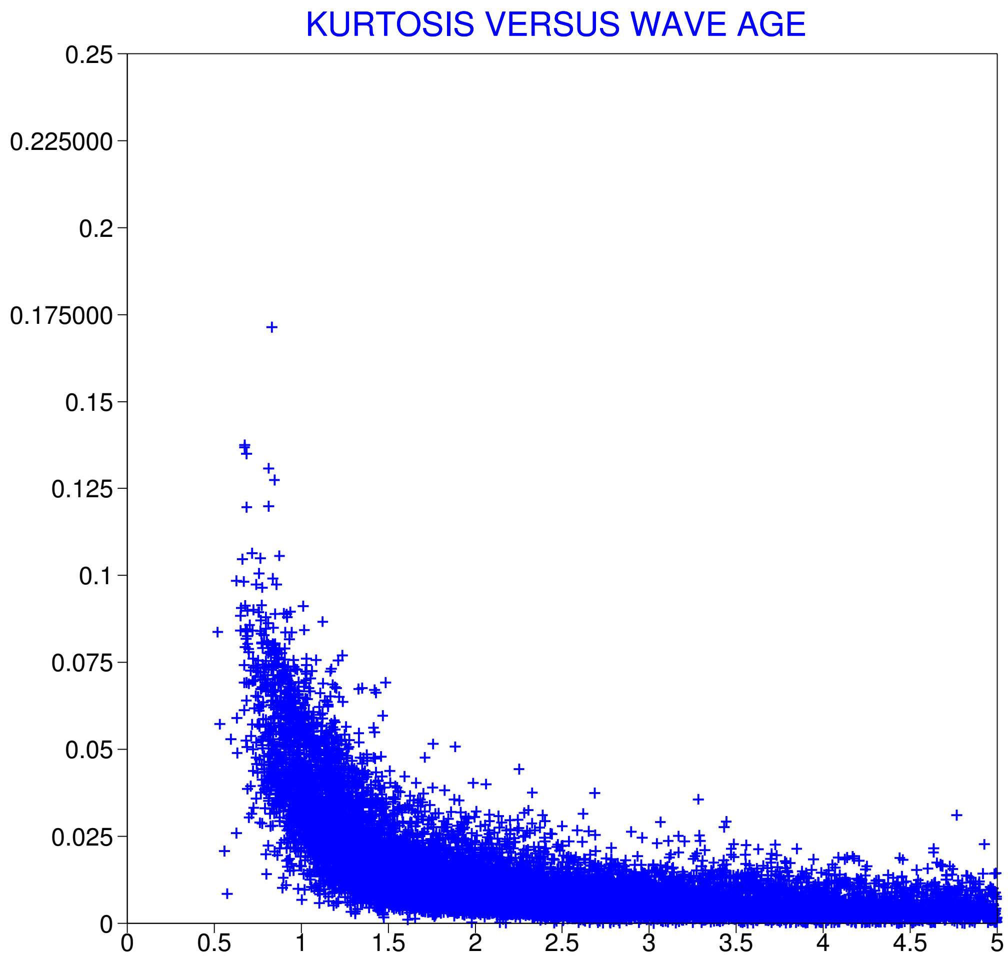
HS an on 2007021000 STEP=00

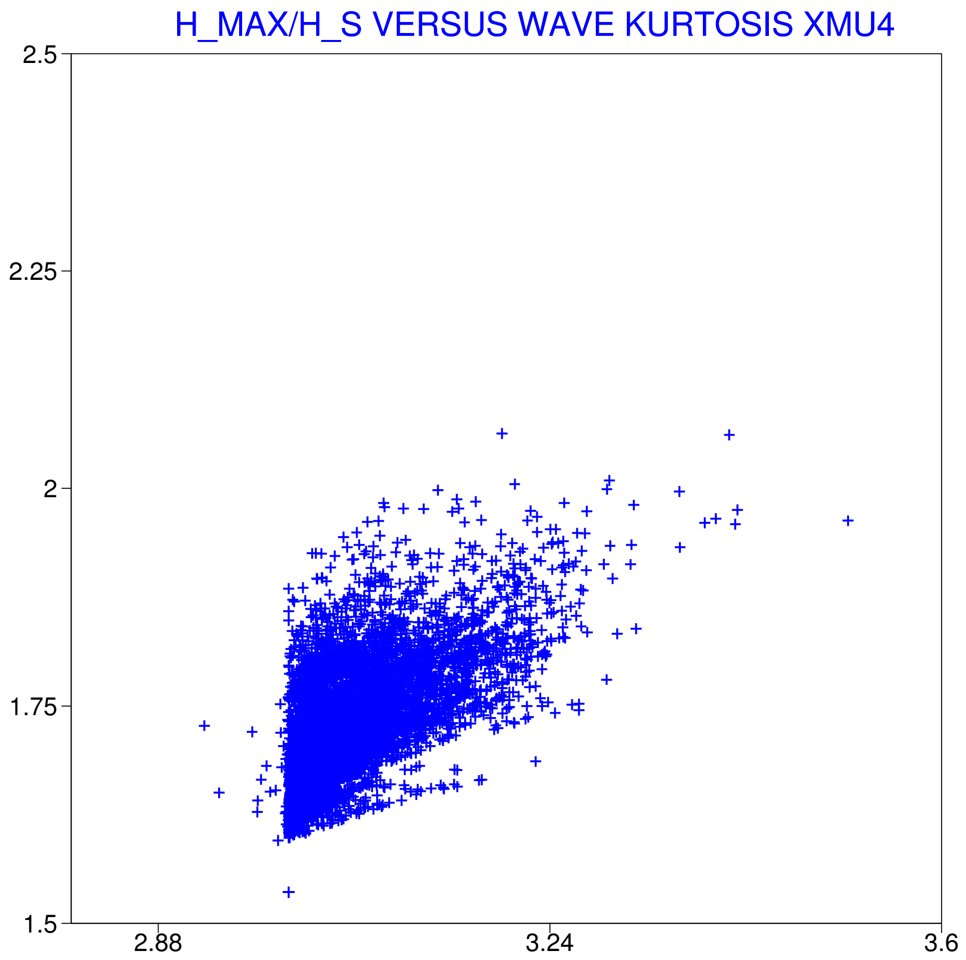



H_MAX an on 2007021000 STEP=00
2.5 - 5
5 - 7.5
7.5 - 10
10 - 12.5
12.5 - 15
15 - 17.5
17.5 - 20
20 - 22.5
22.5 - 25
25 - 26.560
10
5

KURTOSIS on 2007021000 STEP=00




KURTOSIS VERSUS WAVE AGE
0.25
0.225000
0.2
0.175000
0.15
0.125
0.1
0.075
0.05
0.025
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


H_MAX/H_S VERSUS WAVE KURTOSIS XMU4
2.5
2.25
2
1.75
1.5
2.88
3.24
3.6

Validation

A systematic validation of this freak wave warning system against individual events has not been performed so far. This will not be easy as one is bound to compare **apples with pears**, as the freak wave warning system produces statements of a statistical nature while individual events are just a random draw from a large ensemble of possibilities.

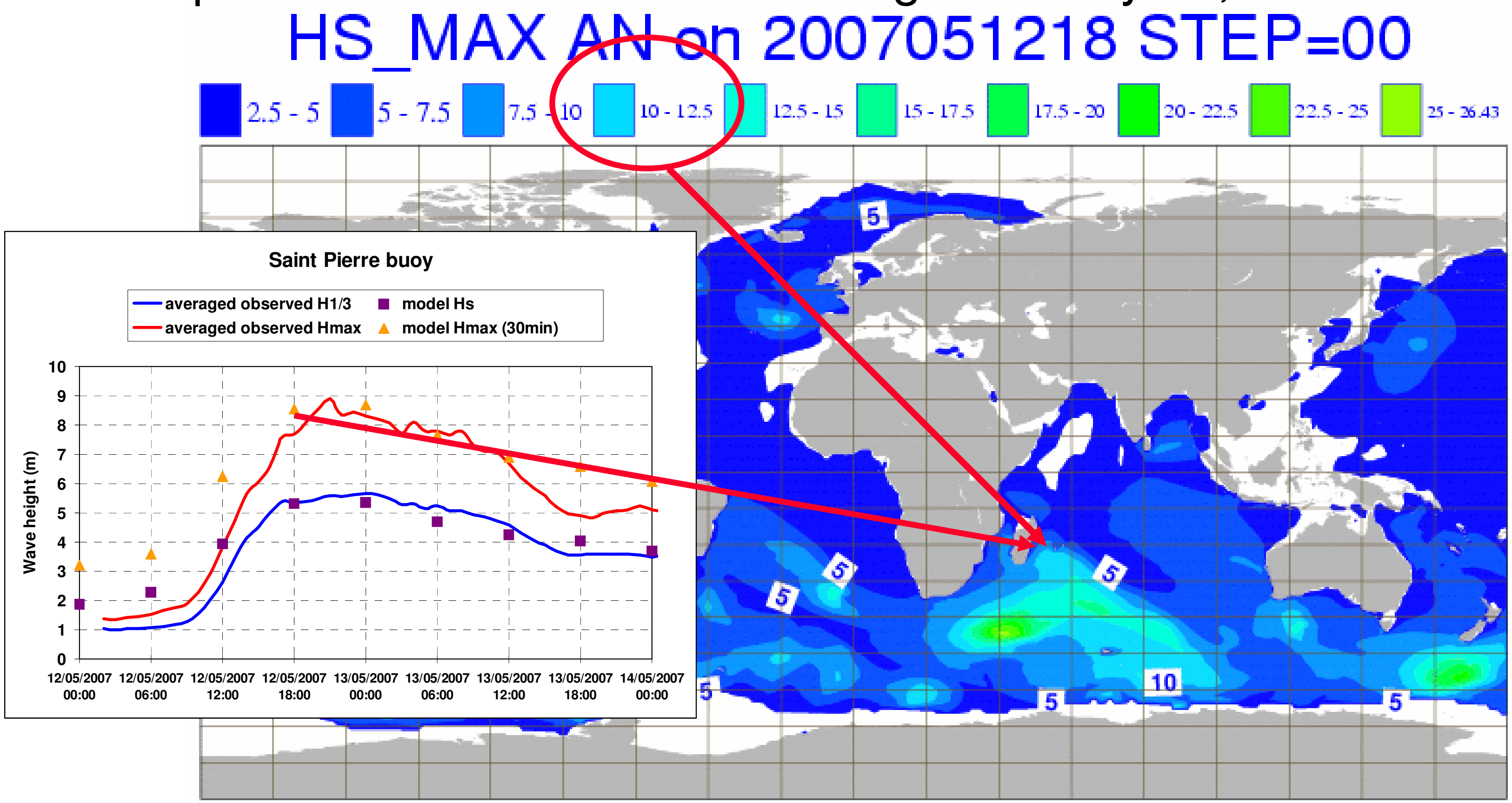
- One case of interest is the La Réunion event, where observations of maximum wave height where averaged over a 4 hour time interval.
- A comparison to check the statistical properties of the extreme wave system was performed by a validation against 5 years of data at AUK which I got from G. Burgers of KNMI.

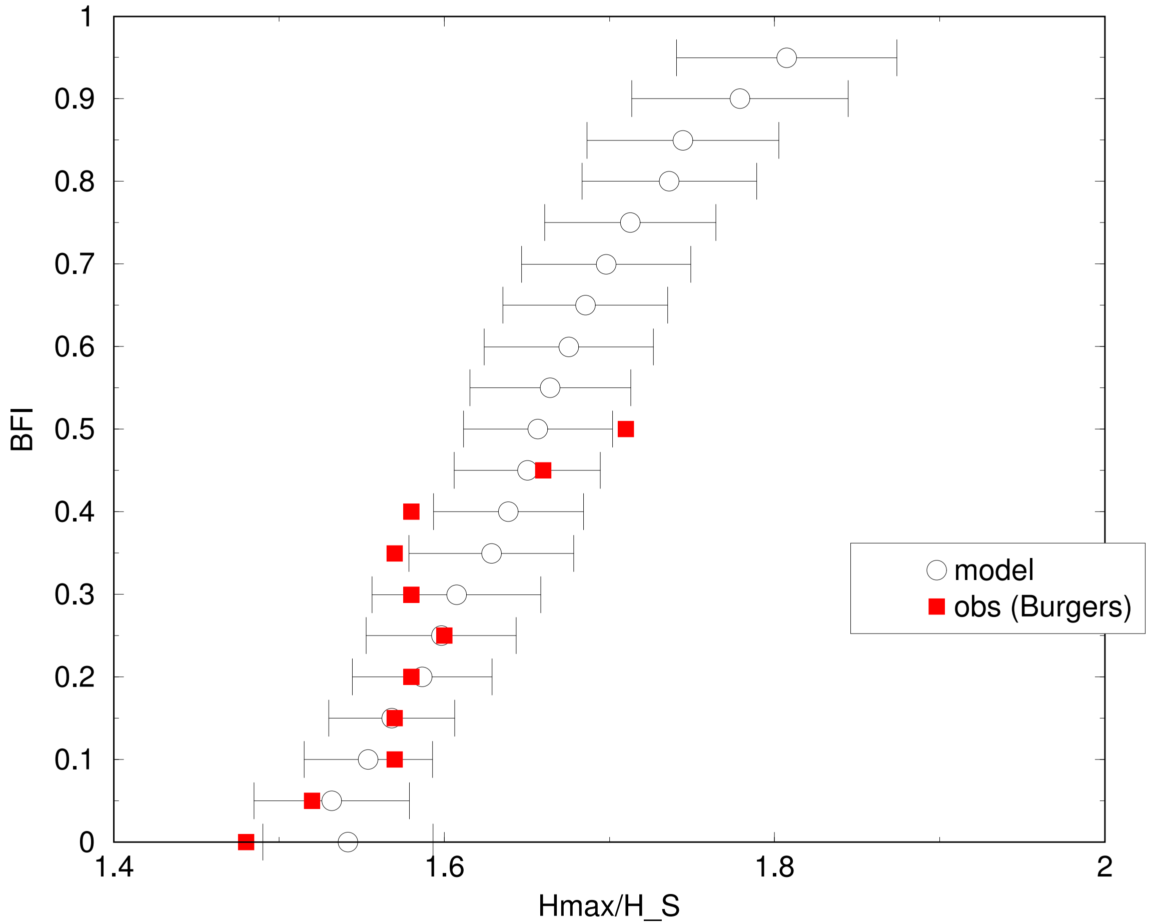
  A reasonable agreement between model and data is obtained.

Large swell reaching la Réunion: the model

A new parameter is being developed. Namely the maximum wave height that can be expected within a certain time window (here 3 hours).
It will be introduced in operations soon.

New parameter: Maximum wave height on May 12, 18UTC




1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
BFI
1.4
1.6
1.8
2
Hmax/H_S
model
obs (Burgers)

CONCLUSIONS

- Presented the extension of the ECMWF freak wave warning system which will be introduced operationally in the beginning of next year. Seems to give realistic results.
- However, the determination of the kurtosis of the wave field is based on the narrow-band approximation, which does not give a truthful description of what happens when two or more nonlinear wave trains are present. In such a case growthrates of the Benjamin-Feir instability are larger, giving larger deviations from Normality. It is highly desirable to obtain a more general algorithm for the kurtosis (say the equivalent of the Direct-Interaction Approximation).
- Validation of this approach in the field is evidently needed. Global satellite data, such as from the Altimeter and the SAR, would be ideal. However, still a lot of work is needed to extract extreme sea state information from these data.