Approaches for the Efficient Probabilistic Calculation of Surge Hazard

by

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Motivation

$$P[\eta_{\max(1\,yr)} > \eta] = \lambda \int \ldots \int_{\underline{x}} f_{\underline{X}}(\underline{x}) P[\eta_m(\underline{x}) + \varepsilon > \eta]\, d\underline{x}$$

Annual rate of storms of interest, within distance range of interest)

Joint probability distribution of storm characteristics (ΔP, Rp, Vf, etc. , landfall location, heading)

$$\underline{X} = (\Delta P, Rp, V_f, \theta, \text{landfall location})$$

Hurricane climatology

Surge effects, given x

- requires wind, wave, & surge calculations for one artificial storm
- **expensive to calculate**
- ε term accounts for errors in numerical surge model and limitations in parameterization

Methodology: 2 JPM-OS approaches

1. Response surface approach
   – Select a set of storms to run (experimental design)
   – Fit simple parametric model to results from runs
   – Evaluation of integral using parametric model (fairly easy because parametric model is very fast)

Methodology (cont'd)

2. Quadrature approach

$$\lambda \int \ldots \int_{\underline{x}} f_{\underline{X}}(\underline{x}) P[\eta_m(\underline{x}) + \varepsilon > \eta] \, d\underline{x} \approx \sum_{i=1}^{n} \lambda_i P[\eta_m(\underline{x}_i) + \varepsilon > \eta]$$

- Approximate multi-dimensional probability distribution by means of a discrete probability distribution
- Set of artificial storms with parameters $\underline{x}_i$ with associated rates λ_i
- Approach: combination of simple and sophisticated numerical integration techniques

Methodology (cont'd)

Notes:

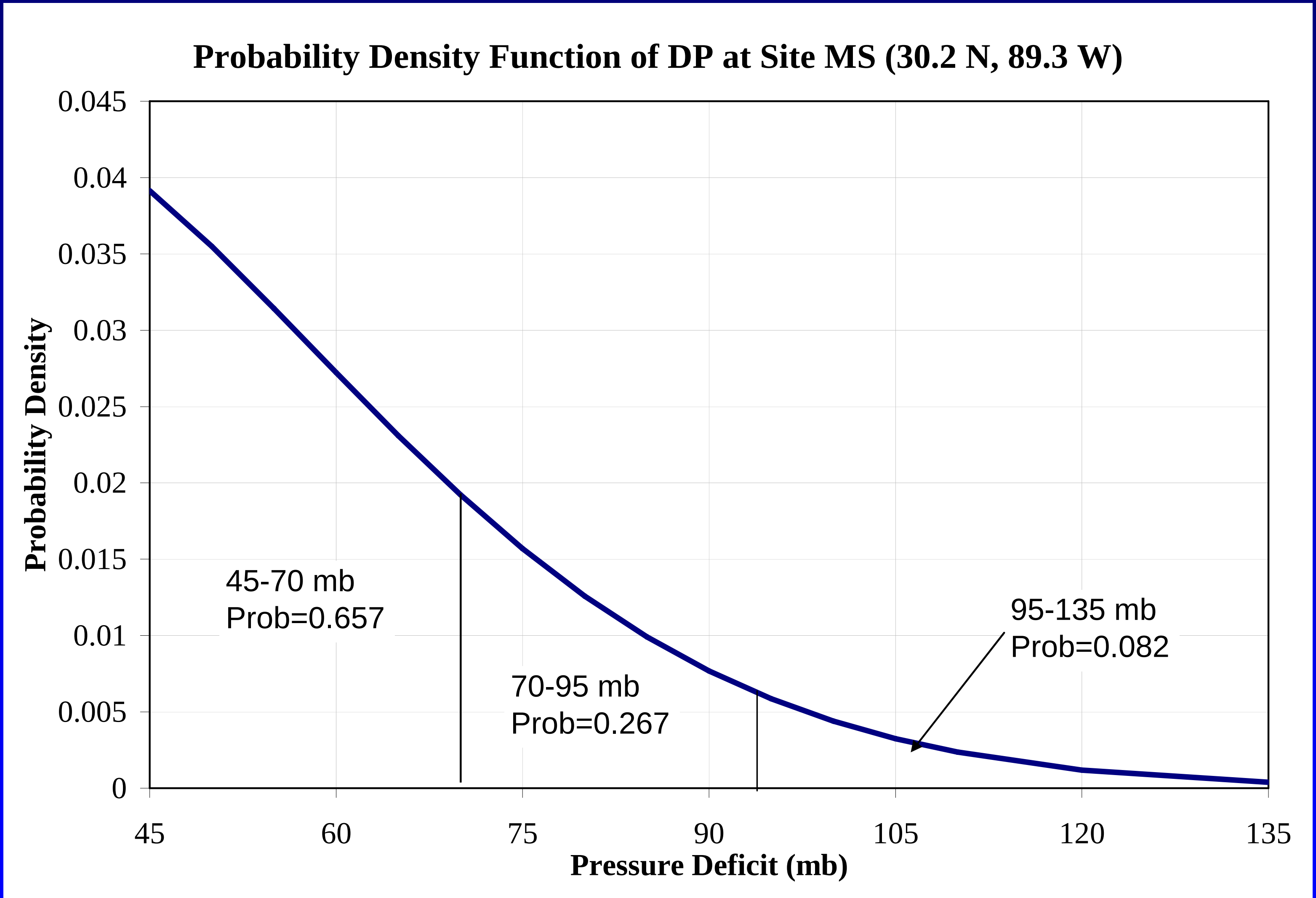
- Both approaches take advantage of the smoothness of $\eta(\underline{\Delta P, R_p, V_f, \theta, location})$
- Quadrature approach assigns weights to the artificial storms, response-surface does not
- In both approaches, final book-keeping (integration) step is straightforward

Conclusions

- 2 JPM-OS methods are available for efficient JPM integration
- Both approaches are practical and have comparable efficiency (< 200 artificial storms to obtain 100- and 500-yr results over 100 km length of coast)
  - Planning side-by-side comparisons
- Need to expand and refine (more realistic hurricane description→more dimensions)

Quadrature JPM-OS: Methodology
(combination of simple and sophisticated numerical integration techniques)

1. Divide probability distribution of ΔP into "slices"
   - Typically 3 slices: roughly corresponding to Cats 3, 4, and 5



Probability Density Function of DP at Site MS (30.2 N, 89.3 W)
Probability Density
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
45
60
75
90
105
120
135
Pressure Deficit (mb)
45-70 mb
Prob=0.657
70-95 mb
Prob=0.267
95-135 mb
Prob=0.082

Quadrature JPM-OS: Methodology (cont'd)

2. For each slice, generate 5-10 combinations of ΔP [within slice], Rp, Vf, Heading taking into account their probability distributions; use *Bayesian Quadrature*

3. Discretize distribution of landfall location using equal spacing *(Rp)* → artificial storms

$I = \int_A f(x)\,p(x)\,dx$

Classical Quadrature (1-D)

$$\int_A f(x)\,p(x)\,dx \approx \sum_{i=1}^{n} w_i\, p(x_i)$$

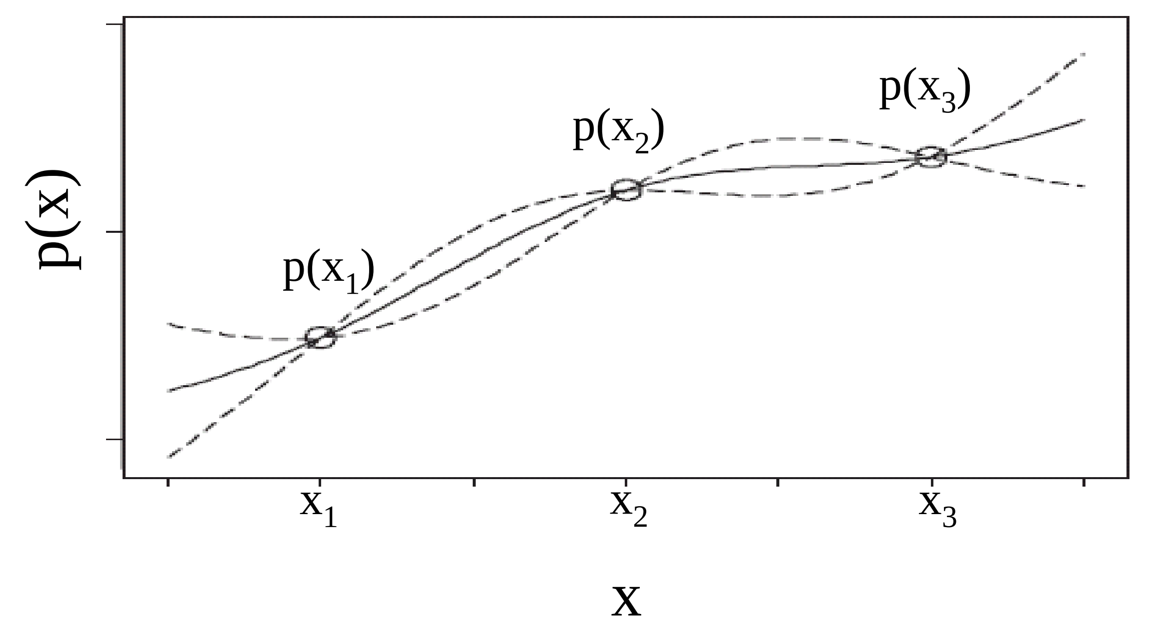
- f(x) is a probability density, p(x) is usually a polynomial of a certain degree
- n, weights w_i and nodal locations x_i determined so that integration error is zero
- Not easy to extend to multiple dimensions in an efficient manner

Bayesian Quadrature

- Represents $p(\underline{x}) = P[\eta_m(\underline{x}) + \varepsilon > \eta]$ portion of integrand as a gaussian random function of $\underline{x}$ with certain correlation properties
- Easy to extend to multiple dimensions
- Key parameter: *correlation distance* in each dimension
  - Focus effort on more important variables by specifying lower correlation distances (guided by sensitivity results)
  - Values are chosen using judgment and then validated using SLOSH

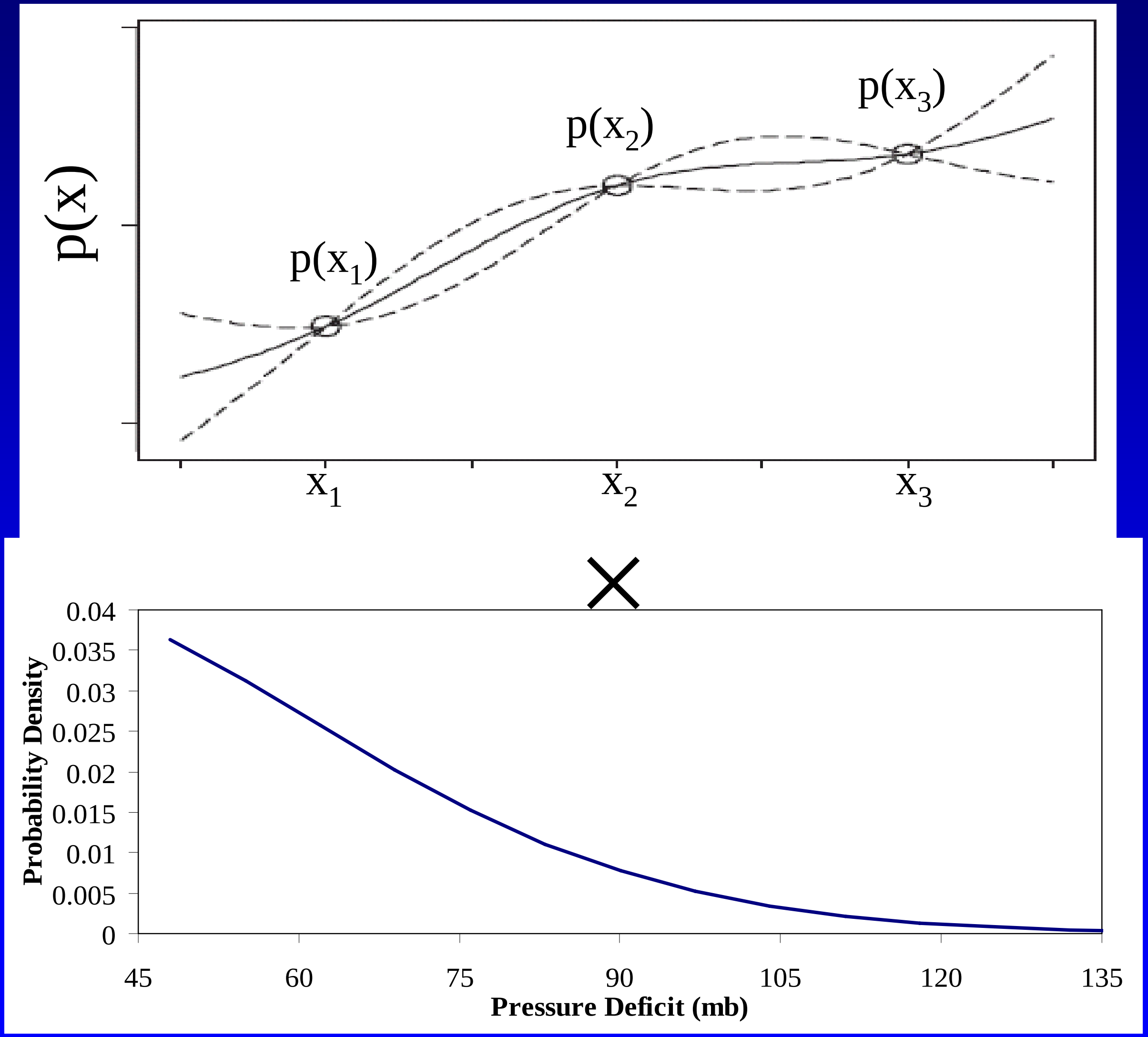
"Bayesian" Quadrature in Detail (Minka's method)

- Think of $p(\underline{x}) = P[\eta_m(\underline{x}) + \varepsilon > \eta]$ portion of integrand as a random function with certain correlation properties



1D example: Have results from 3 artificial storms

What we want: integral of product



Optimization: 2 nested loops

– Inner loop: for given locations of x_1, x_2, x_3, …, find optimal weights that minimize variance of

Exact Integral – Weighted Sum

(analogous to "Kriging"; not too different from least-squares regression)

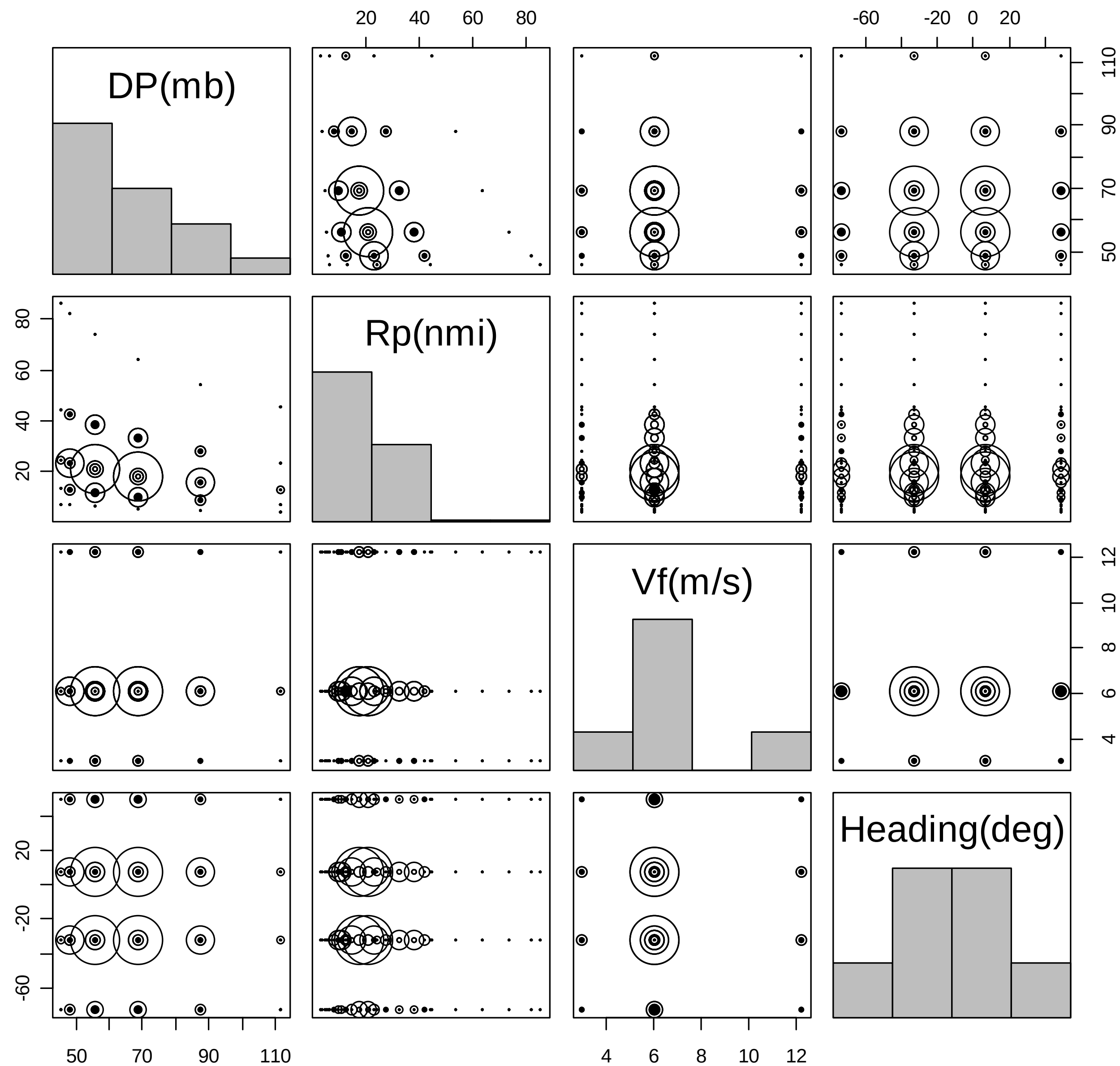
- Outer Loop: find optimal locations of x_1, x_2, x_3, …. to minimize variance; use a derivative-free algorithm (Powell's NEWUOA)

Validation (using SLOSH)

Reference case: JPM-Heavy (Gold Standard)

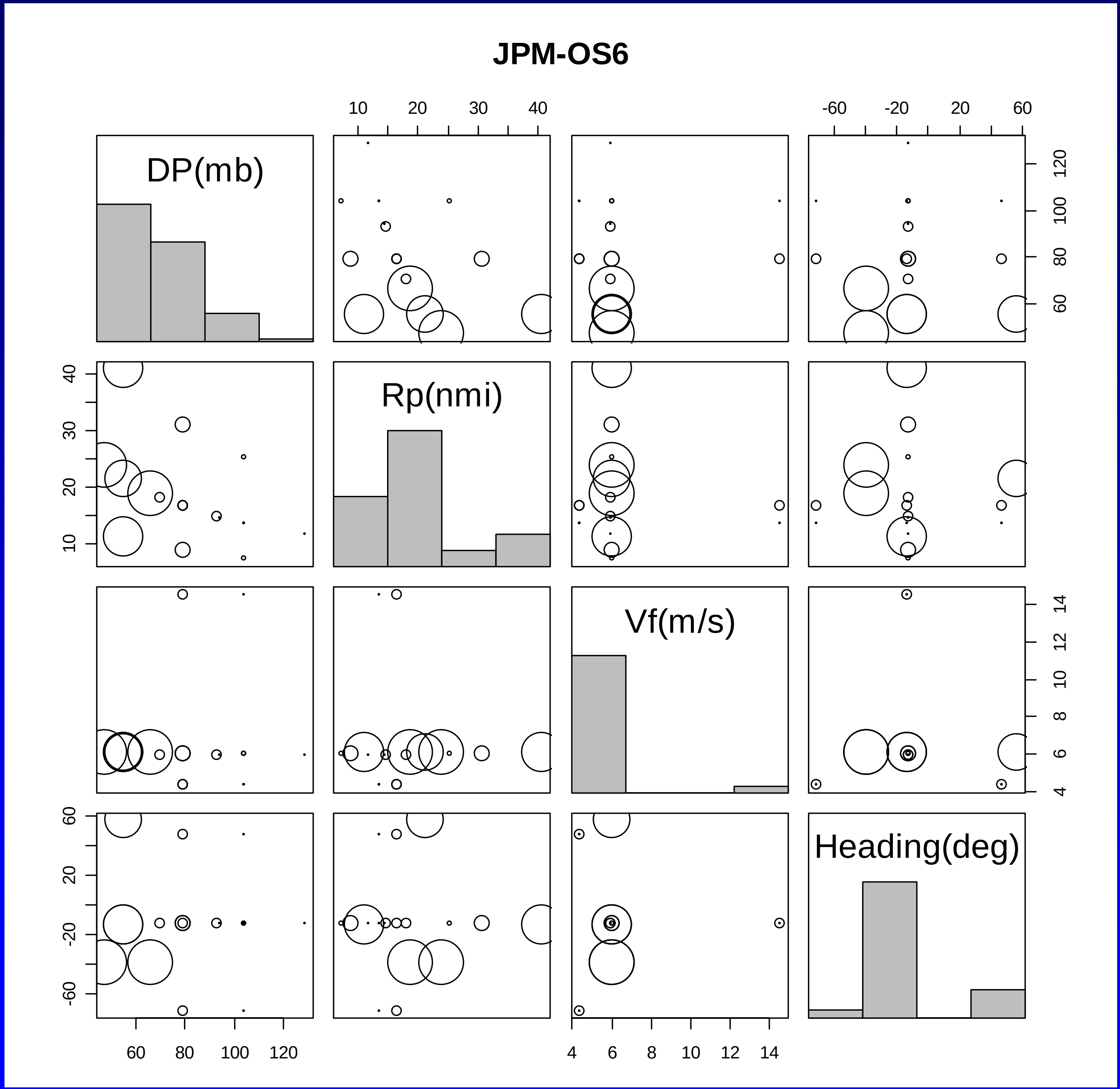
- Discretize distributions of storm parameters:
  - 6 ΔP values
  - 5 $Rp|\,\Delta P$ values
  - 4 headings
  - 3 fwd. velocity values
  - Locations: Rp spacing
- All combinations: 2,967 artificial storms

JPM-Heavy (Gold)



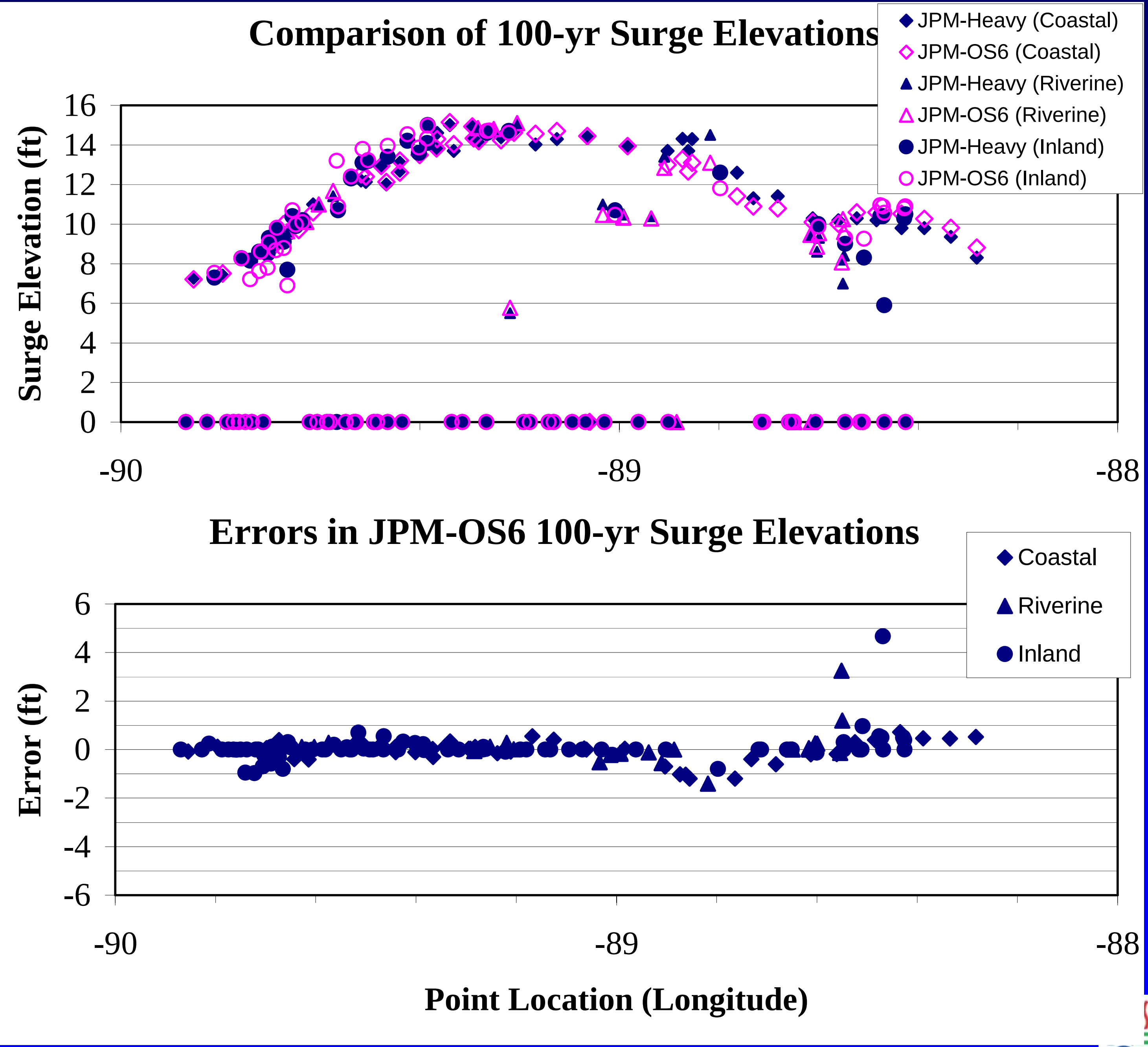
240 combinations of DP, Rp, Vf, θ;

2,967 artificial storms



19 combinations of DP, Rp, Vf, θ;

147 artificial storms



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Conclusions

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- Both approaches are practical and have comparable efficiency (< 200 artificial storms to obtain 100- and 500-yr results over 100 km length of coast)
  - Planning side-by-side comparisons (SLOSH? ADCIRC with simpler grid?)
- Need to expand and refine (more realistic hurricane description→more dimensions)