

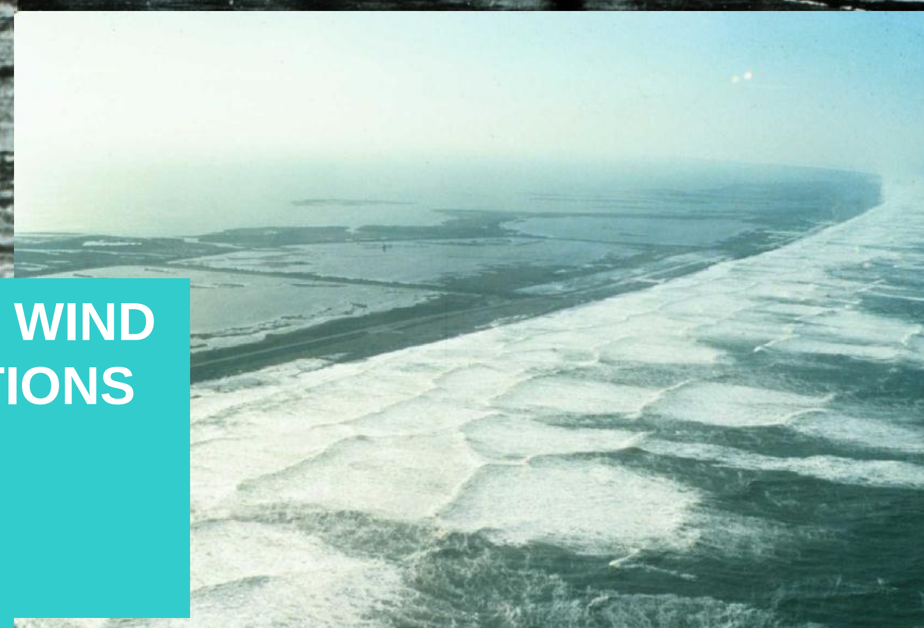

THE HIGH-FREQUENCY PORTION OF WIND WAVE SPECTRA AND ITS IMPLICATIONS

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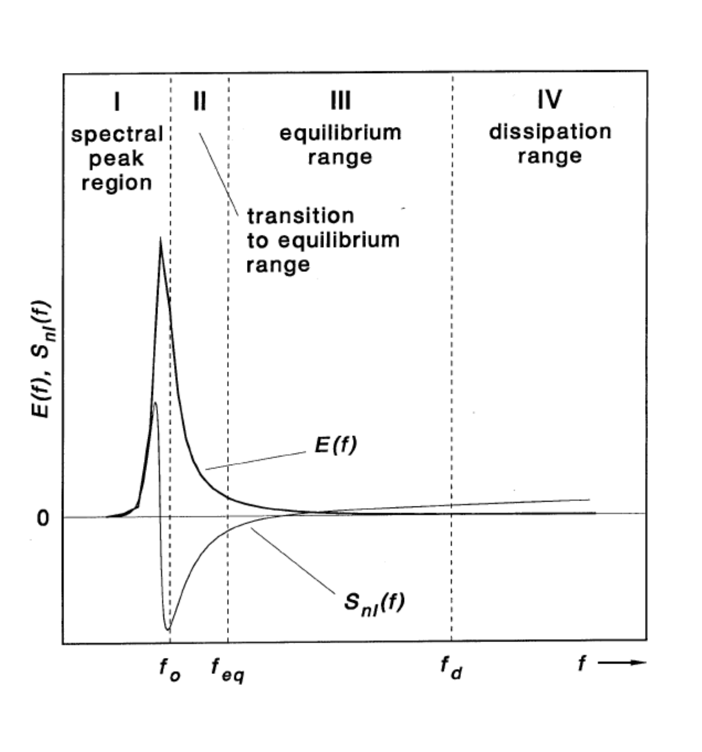
MOTIVATION

Spectral shape provides critical information for understanding source term balance in wind wave spectra



Example isEquilibrium Range forms:

Phillips, Kitaigorodksii (1958-1960's)
$E(f) \sim f^{-5}$

Toba, Donelan, Belcher,etc. (1974-1990's)
$E(f) \sim uf^{-4}$

Resio, Long & Vincent (2004-2001)
$E(f) \sim (u^2 c_p)^{1/3} f^{-4}$

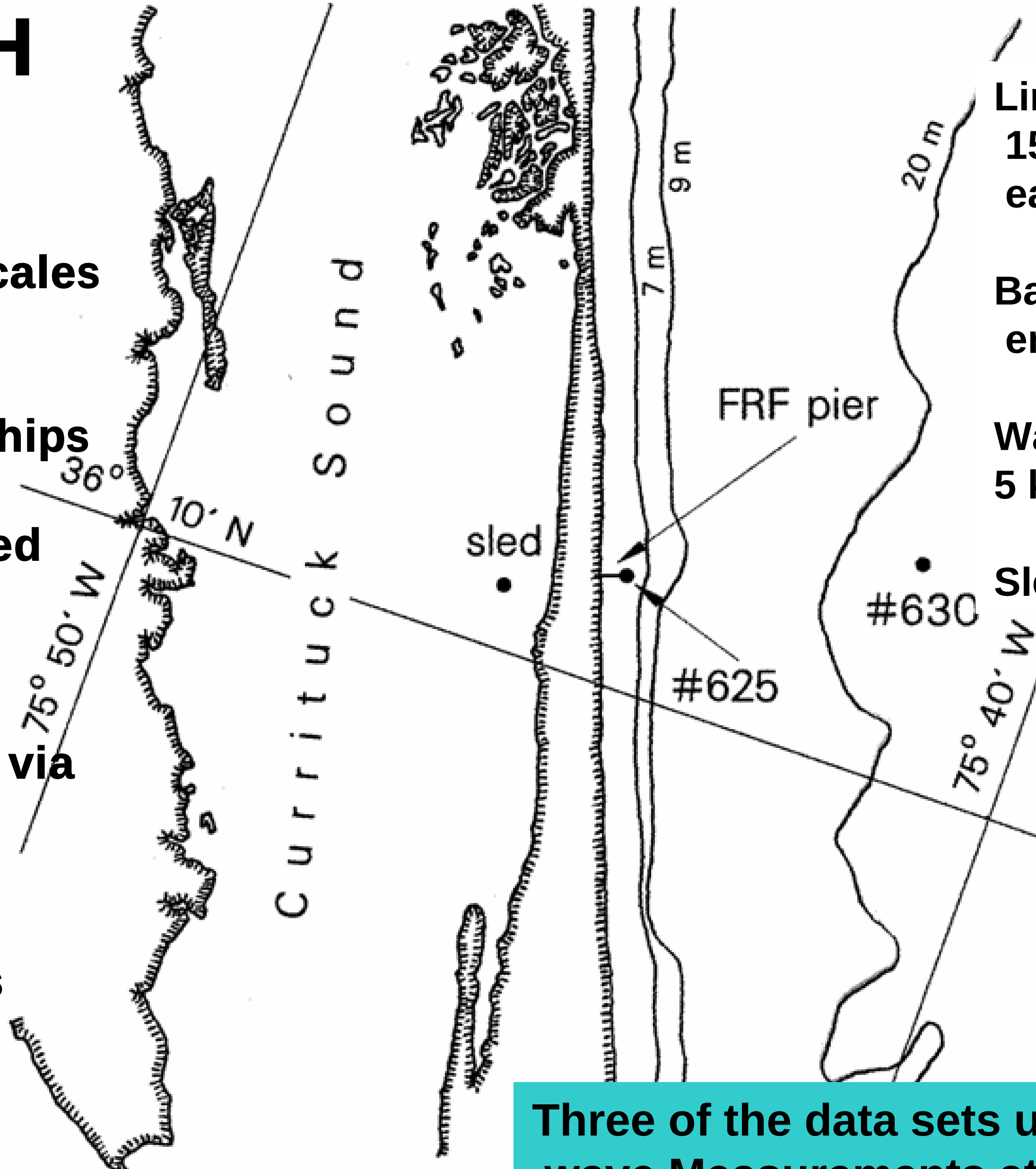
Forristall; Long & Resio (1981-2007)
$F(f) \sim f^{-4} \longrightarrow f^{-5}$

Impacts:

- **Air-sea interaction**
- **Model source terms**
- **Wave set up**

APPROACH

- Use data from a wide range of scales
- Develop physical scaling relationships
- Compare expected scaling behavior with data sets
- Interpret results via individual wave kinematics
- Examine impacts on other area



Linear array about 150 m north and east of pier end

Baylor gage at end of pier

Waverider about 5 km off coast

Sled in the Sound

Three of the data sets use are wave Measurements at Duck, NC

CONCLUSIONS

- Spectra consistently transition from equilibrium range to an f^{-5} form at high frequency in all data sets
- Transition location is consistent with a balance between nonlinear fluxes and high-frequency dissipation
- Kinematic constraints suggest little or no breaking in the spectral peak region
- High-frequency breaking primarily due to local accelerations at high frequencies or orbital velocities exceeding c/2
- This has very significant implications for wave model source terms, air-sea interactions, and wave set-up at coast

Phillips, 1958 $E(f) \sim \alpha_5 g^2 f^{-5}$

Toba, 1974 $E(f) \sim \alpha_4 u g f^{-4}$

where

α_4 is the equilibrium range coefficient and

u is term with units of velocity.

Resio, Long, & Vincent 2001 $E(f) : \alpha_4 (u^2 c_p)^{1/3} f^{-4}$

Where
C_p = phase velocity of spectral peak

Forristall, 1981 $E(f) \sim \alpha_4 u g f^{-4} \quad \rightarrow \quad \sim \alpha_5 g^2 f^{-5}$

for $\hat{f} \ (= u f g^{-1}) > const.$

Switching to wavenumber spectral basis

$$F(k) = \frac{\beta}{\sqrt{g}} k^{-5/2}$$

where

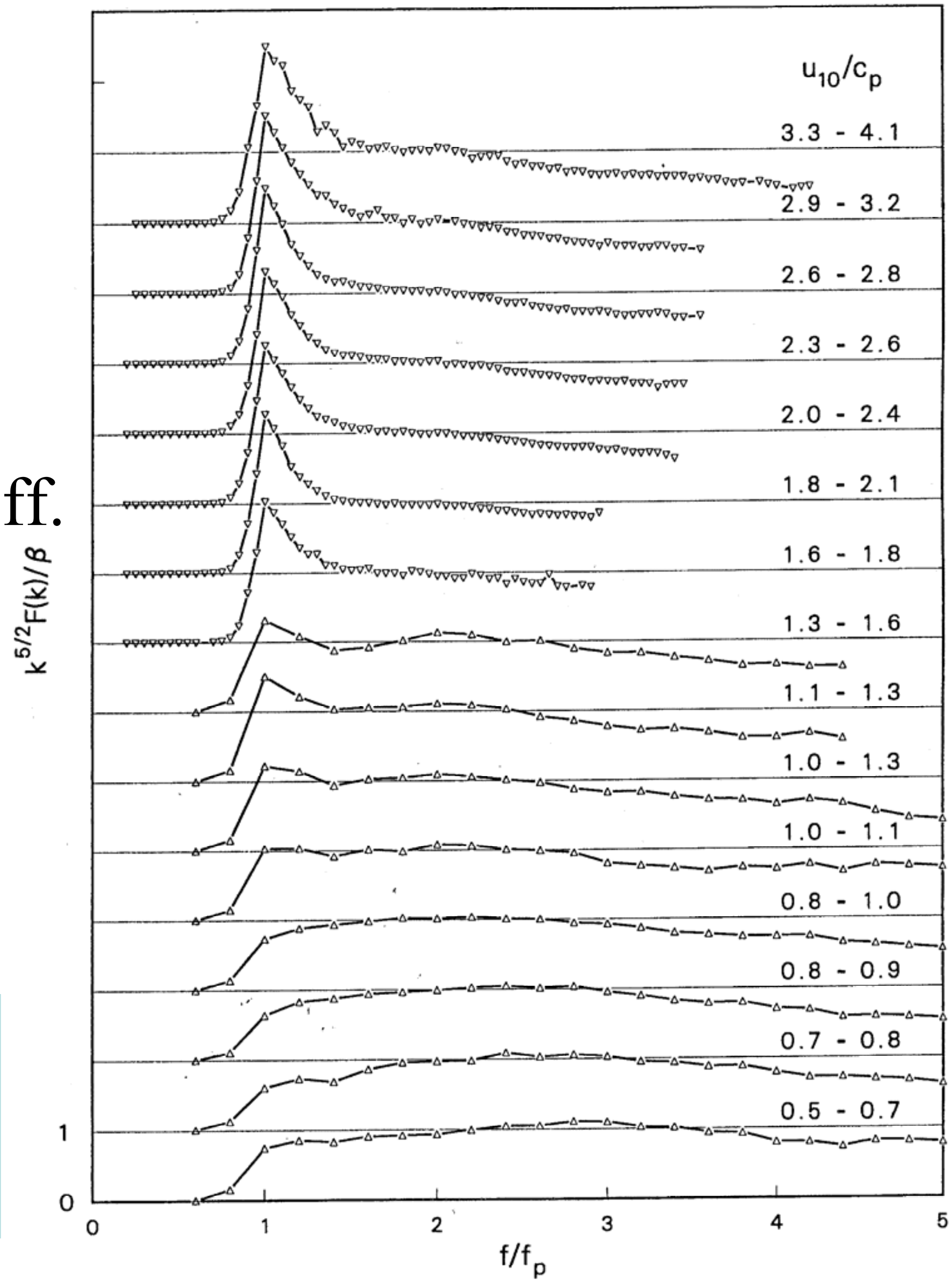
β is the equilibrium range coeff.

k is wavenumber.

In deep water:

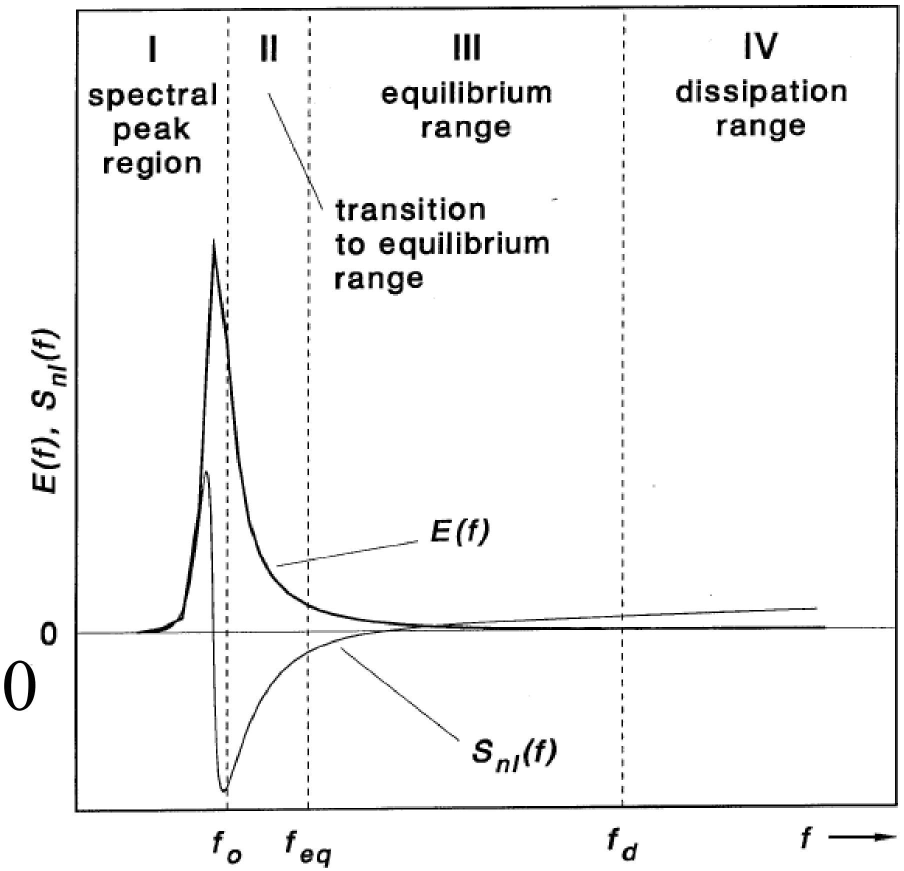
$$\beta = \tfrac{1}{2}\alpha_4 u g^{-1/2}$$

Equilibrium form appears to transition to different form at high frequencies



Simple idea of energy and momentum balance

(similar to Hasselmann et al 1973)



In equilibrium range:

$$\frac{\partial \Gamma_E}{\partial f} = \sum (S_{in}(f) + S_{ds}(f)) \approx 0$$

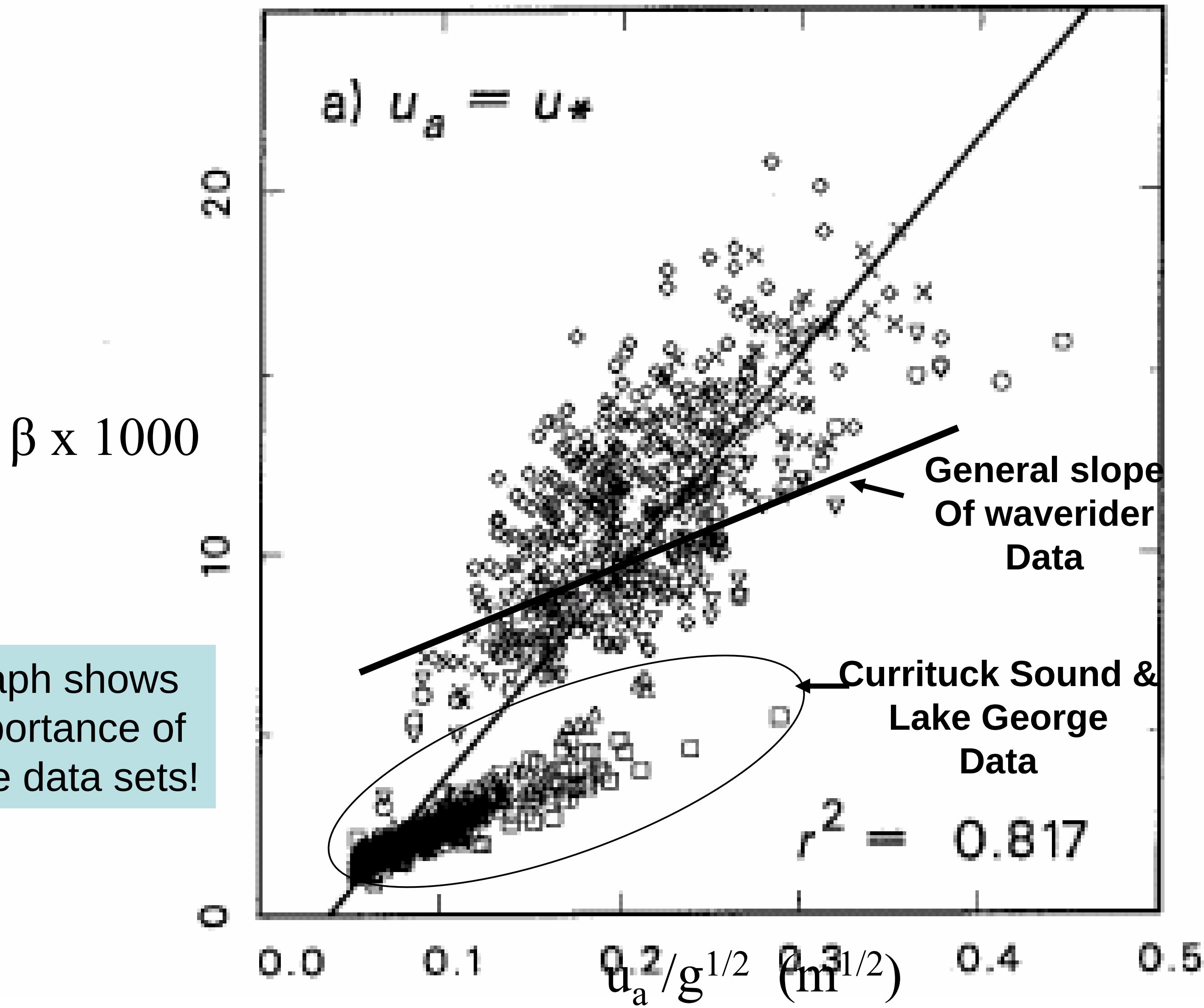
where

Γ_E is the net flux of energy through the spectrum

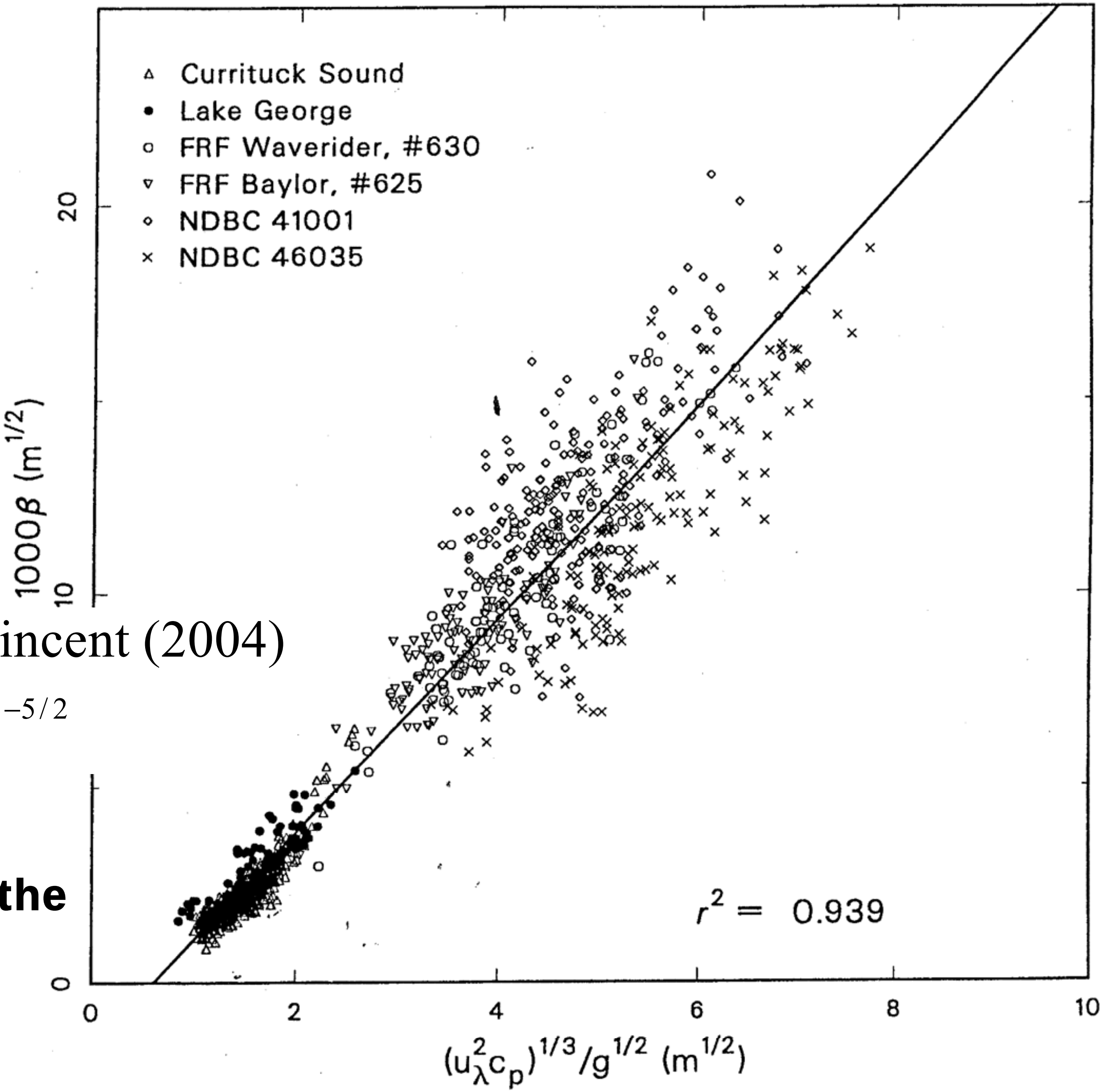
$S_{in}(f)$ is the wind input at frequency f,

$S_{ds}(f)$ is the dissipation sink at frequency f.

Toba, Belcher and others have postulated that β is linearly proportional to wind speed. This clearly does not work for multiple data sets.



This graph shows the importance of multiple data sets!



Resio, Long & Vincent (2004)

$$F(k) : (u^2 c_p)^{1/3} k^{-5/2}$$

But where does the transition to a high-frequency form occur?

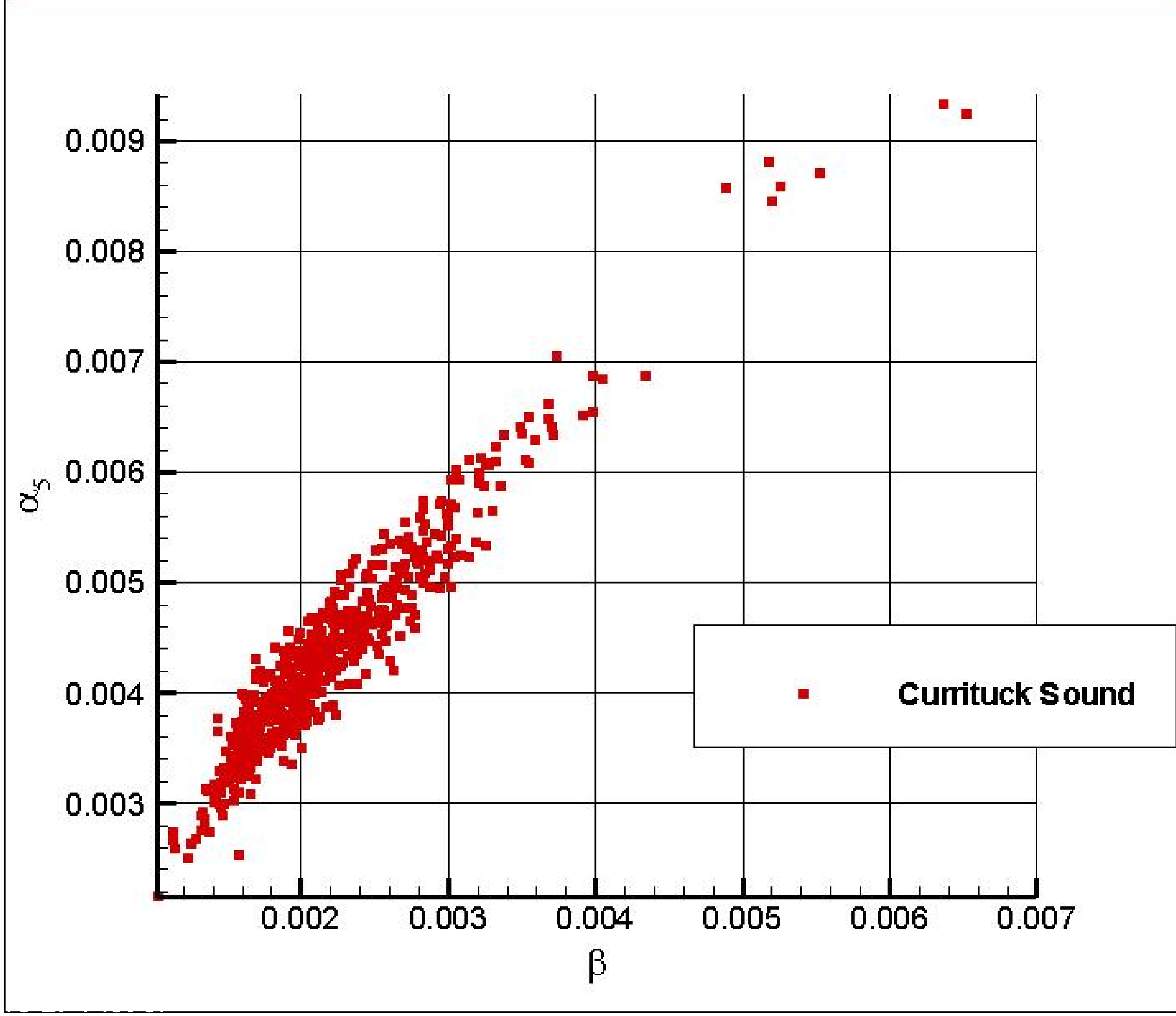
Hypothesis: If wave breaking is confined primarily to a high-frequency range then the energy loss rate in this high-frequency zone must balance the Flux of energy toward high frequencies in equilibrium range – given by

$$\Gamma_E = \frac{\Lambda \beta^3}{g}$$

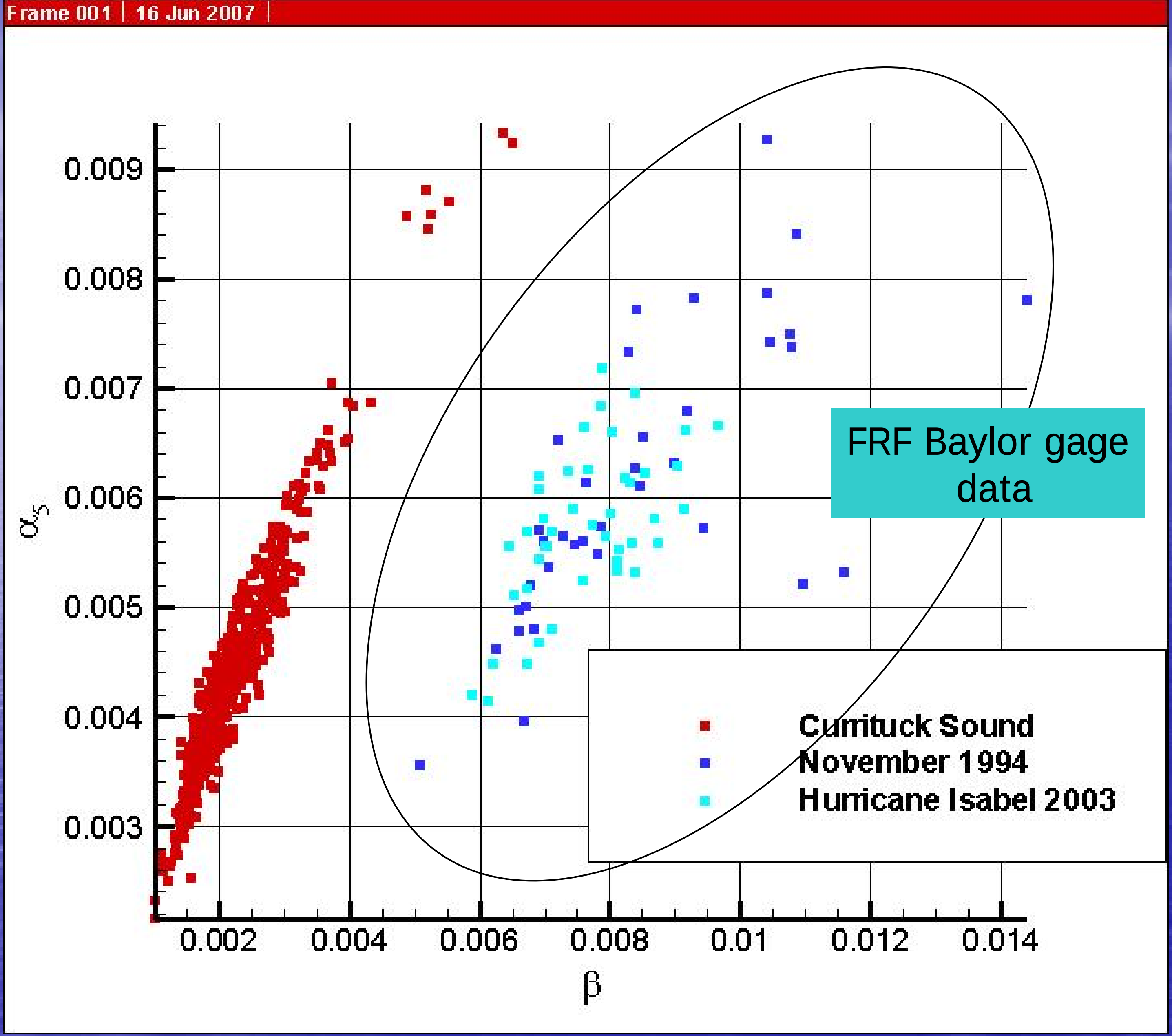
where Λ is a nearly constant dimensionless coefficient.

Data Sources to examine this hypothesis:

1. **Currituck Sound – capacitance wave array;**
2. **Field Research Facility (FRF) – Baylor gauge;**
3. **Field Research Facility (FRF) – Waverider buoy;**
4. **WACSIS – Baylor gauge; and**
5. **WACSIS – Waverider buoy.**



Plot of alpha v. beta for Currituck Sound data only



Plot of alpha versus beta for three different sets of wave spectra. There is an obvious scaling difference between the ocean-scale spectra and the Currituck spectra.

For a k^{-3} spectral tail, the energy lost when a wave at wavenumber k_b breaks:

$$\Delta E_L = \frac{\lambda_b \alpha_5 k_b^{-2}}{2}$$

where

ΔE_L is the energy lost when a wave at wavenumber k_b breaks, and

λ_b is the proportion of the energy that is lost in a single breaking event.

$$\alpha_5 k_x^{-3} \sim \beta g^{-1/2} k_x^{-5/2} \longrightarrow \alpha_5 \sim \frac{\beta}{c_x}$$

$$\Gamma_{ds} \sim \Delta E_L < f_b >$$

where

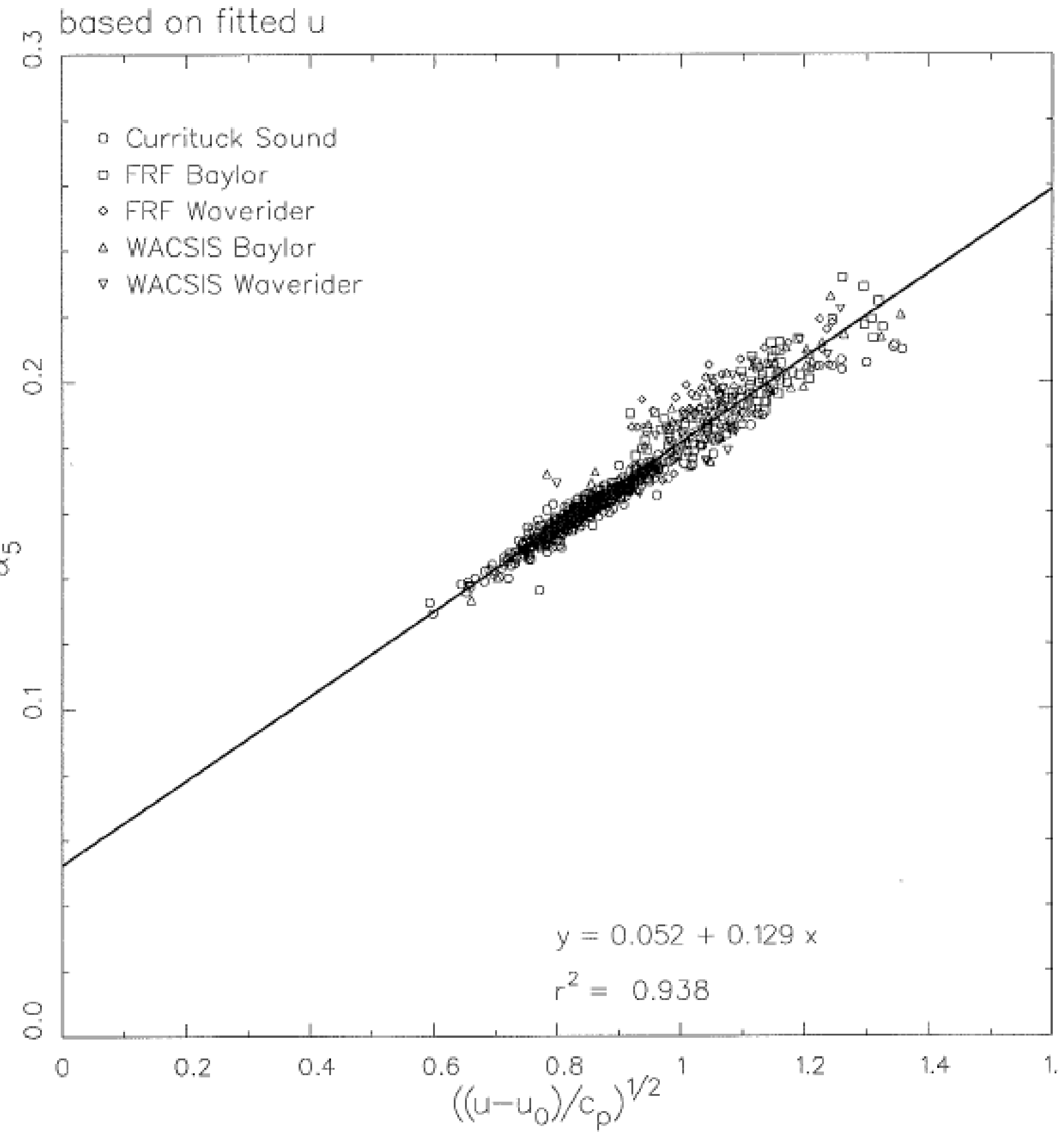
$< f_b >$ is the average frequency of breaking.

For transition frequency in relatively deep water:

$$\Gamma_{ds} \sim \alpha_5 k_x^{-3/2}$$

$$\alpha_5 \sim \left(\frac{\beta}{c_x}\right)^3$$

$$\alpha_5^{1/3} \sim \left(\frac{\beta f_p}{g}\right)^{1/2} \sim \left(\frac{\beta}{c_p}\right)^{1/2} \sim \left(\frac{u - u_0}{c_p}\right)^{1/2}$$



based on fitted u
Currituck Sound
FRF Baylor
FRF Waverider
WACSIS Baylor
WACSIS Waverider
y = 0.052 + 0.129 x
r^2 = 0.938
$\alpha_5^{1/3}$
$((u-u_0)/c_p)^{1/2}$
0.0
0.1
0.2
0.3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

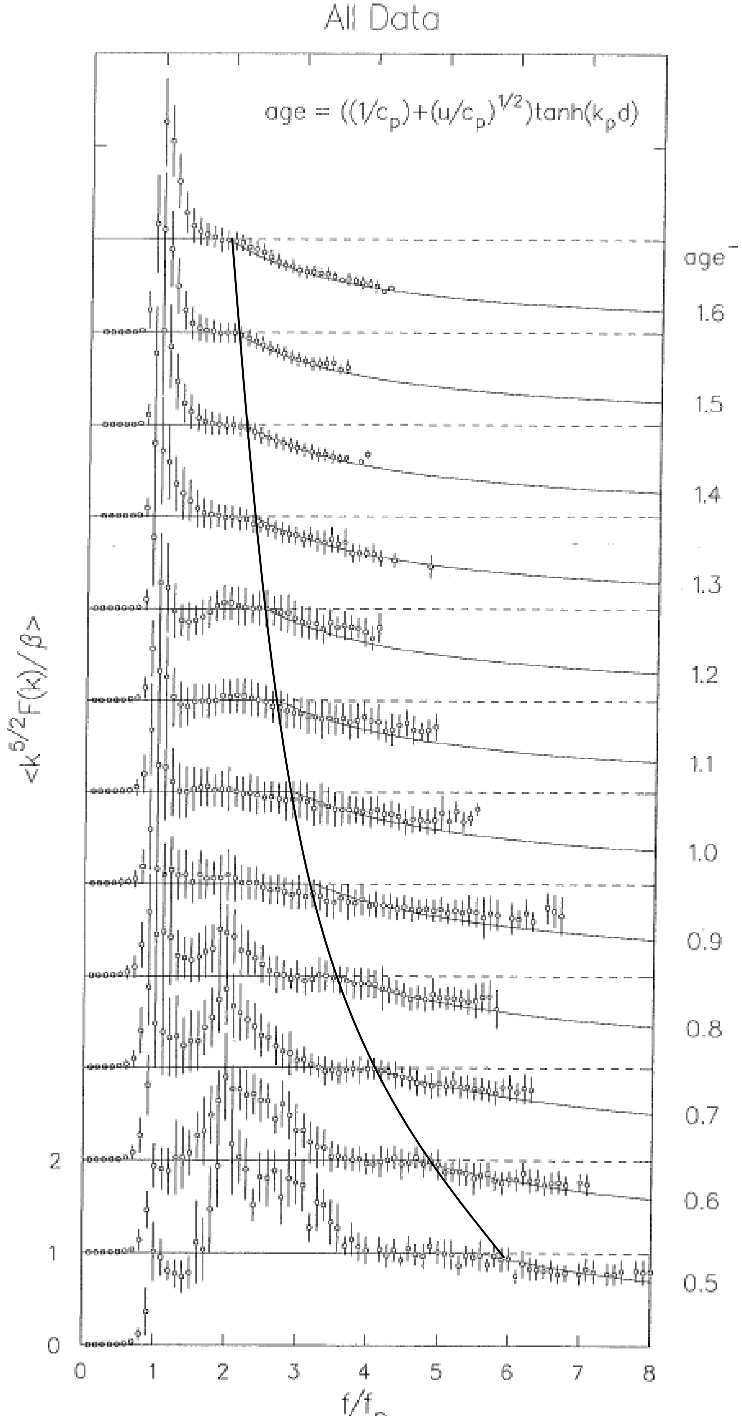
Some algebra shows that we should have the transition relative to the spectral peak frequency at a location that varies with the square root of $\frac{c_p}{\beta}$:

$$\hat{f}_t = \frac{f_t}{f_p} \sim \sqrt{\frac{c_p}{\beta}}$$

where

$\hat{f}_t$ is the relative frequency of the transition. Note that Forristall (1981) implies

$$f_t \sim \frac{c_p}{\beta}$$



Traditional spectral random phase simulation in time domain:

$$\eta(t) = \sum_{k}^{\#angles} \sum_{j=1}^{\#frequencies} a_{j,k} \cos(\omega_j t + \phi_{j,k})$$

$$u_x(t) = \sum_{k}^{\#angles} \sum_{j=1}^{\#frequencies} \omega_j a_{j,k} \cos(\omega_j t + \phi_{j,k}) \cos(\theta_k)$$

$$\ddot{v}(t) = \sum_{k}^{\#angles} \sum_{j=1}^{\#frequencies} \omega_j^2 a_{j,k} \cos(\omega_j t + \phi_{j,k})$$

with

$$a_{j,k} = \sqrt{E(f,\theta)\delta f \delta\theta}$$

where

$\eta(t)$ is the wave surface elevation at time t,

$a_{j,k}$ is the amplitude of the jth frequency and kth angle region of the spectrum

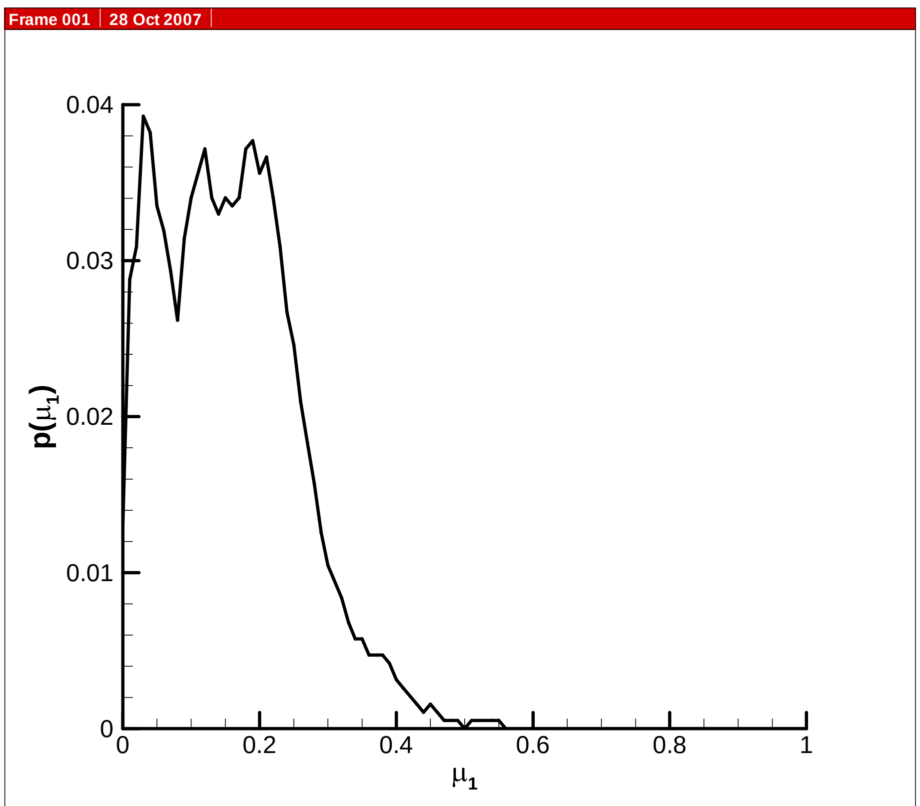
$u_x(t)$ is the horizontal velocity in the x direction at time t,

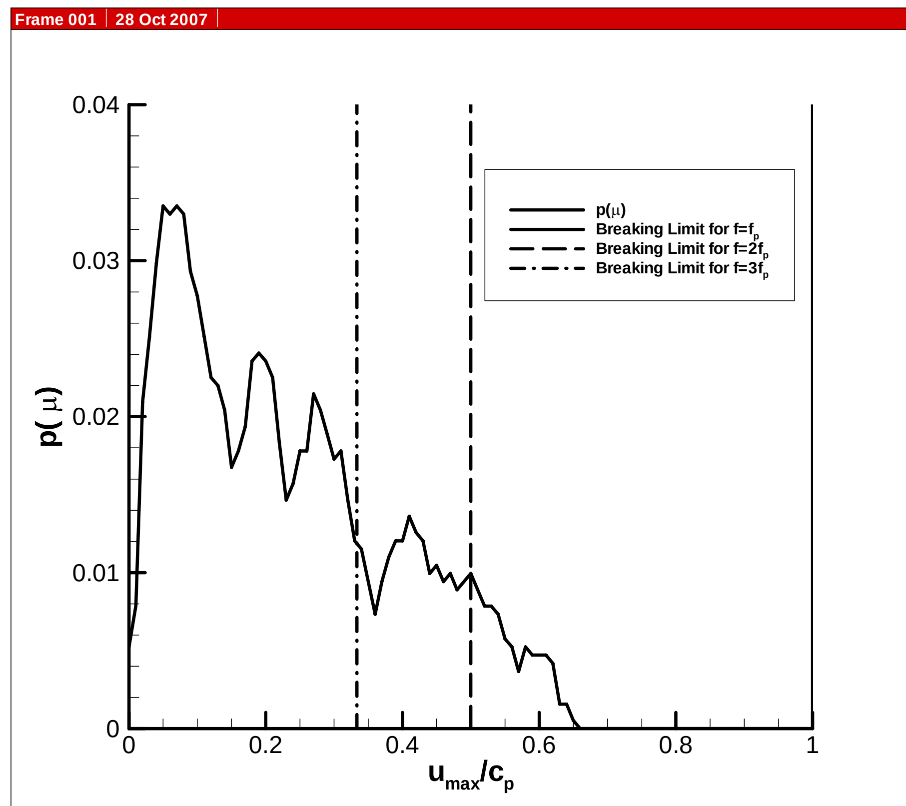
$\ddot{v}(t)$ is the vertical acceleration of the water surface at time t, and

$\delta f \delta\theta$ is the discrete bands of frequency and angle used in the simulation.

Three constraints from "individual" wave kinematics:

1. wave steepness $\mu_1 = H / L < 1/7$
2. acceleration $\mu_2 = \dot{w}/(g/2)$
3. ratio of orbital velocity to phase velocity $\mu_3 = u_{max} /(2c)$



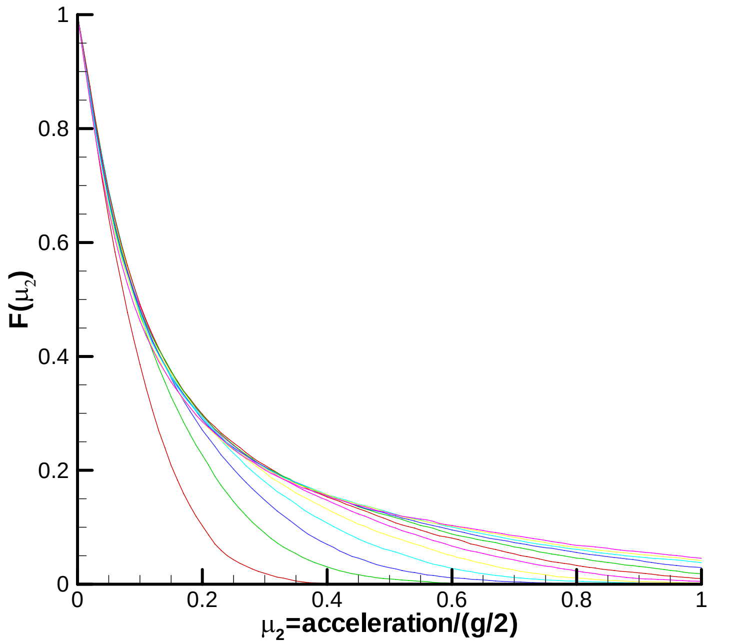


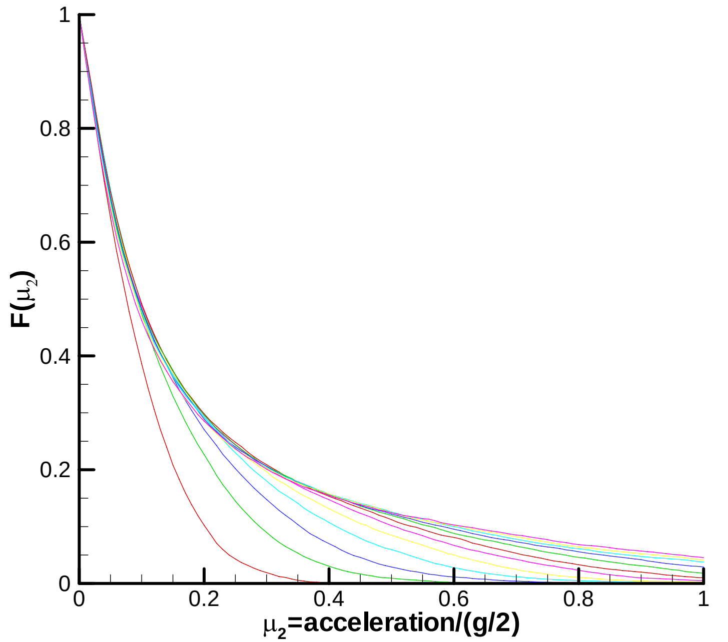
Statistical framework for acceleration breaking limit:

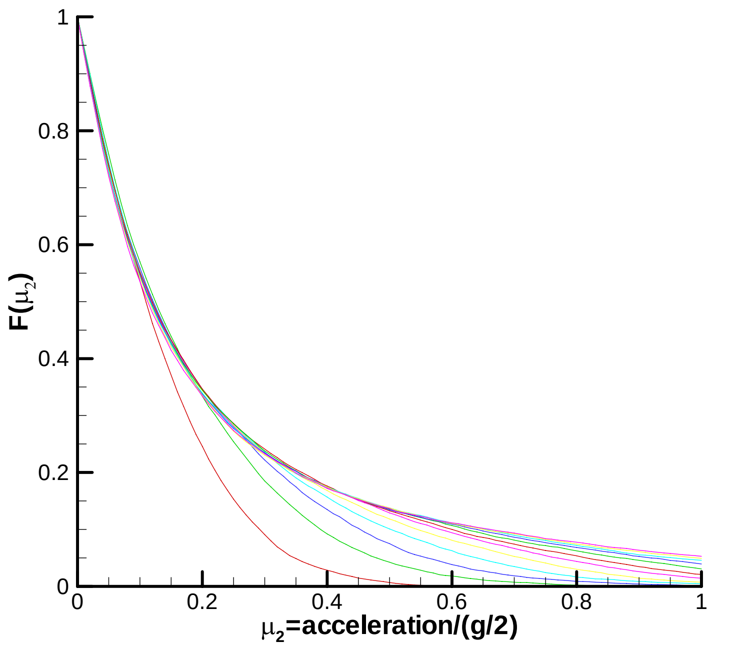
$$< a_b >= Q_1 \int_0^{f_{eq}} f^4 f^{-4} \phi(f / f_p) df + Q_2 \int_{f_{eq}}^{f_t} f^4 f^{-4} df + Q_3 \int_{f_t}^{f_{cap}} f^4 f^{-5} \phi(f / f_p) df$$

$$\frac{\partial < a_b >}{\partial f_t} = Q_1 \frac{\partial \phi(f / f_p)}{\partial f_t} + Q_2 f_t + \frac{Q_3}{f_t}$$

From this representation we see that the f^{-4} tail cannot extend too high or the accelerations will become very large.







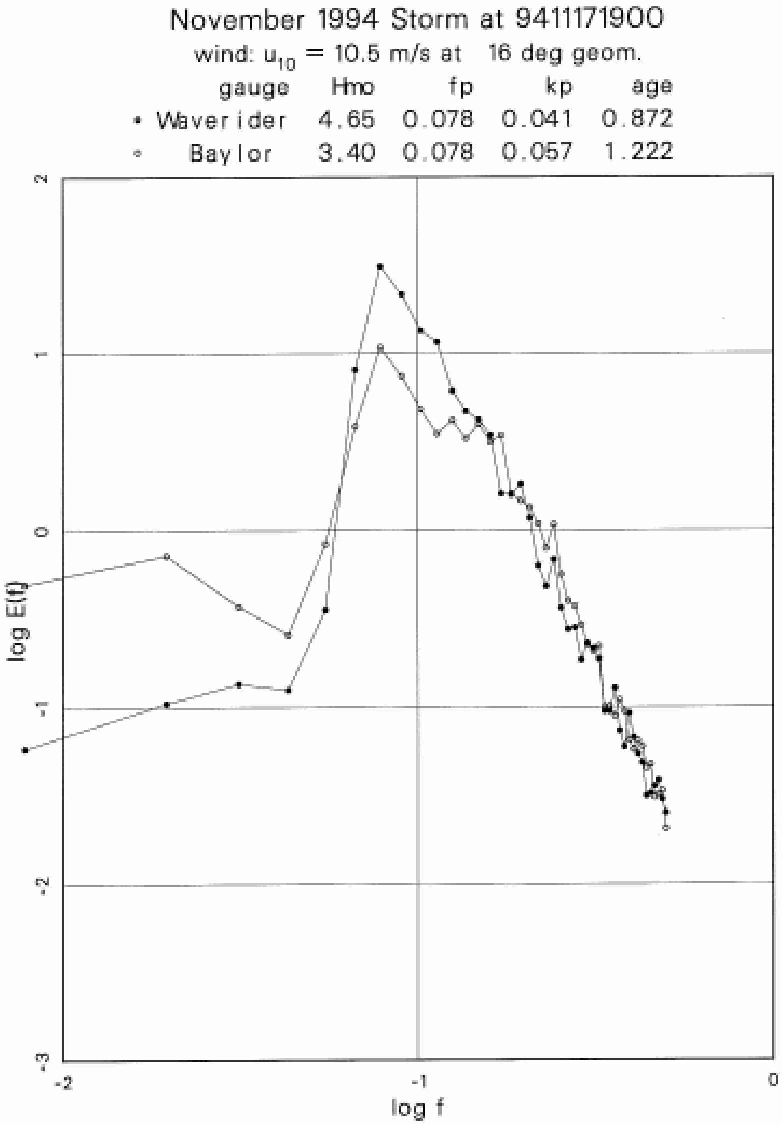
Cumulative Distribution Function for normalized acceleration as a function of relative peakedness for a range of upper frequencies from 1.5 – 6.0 f_p

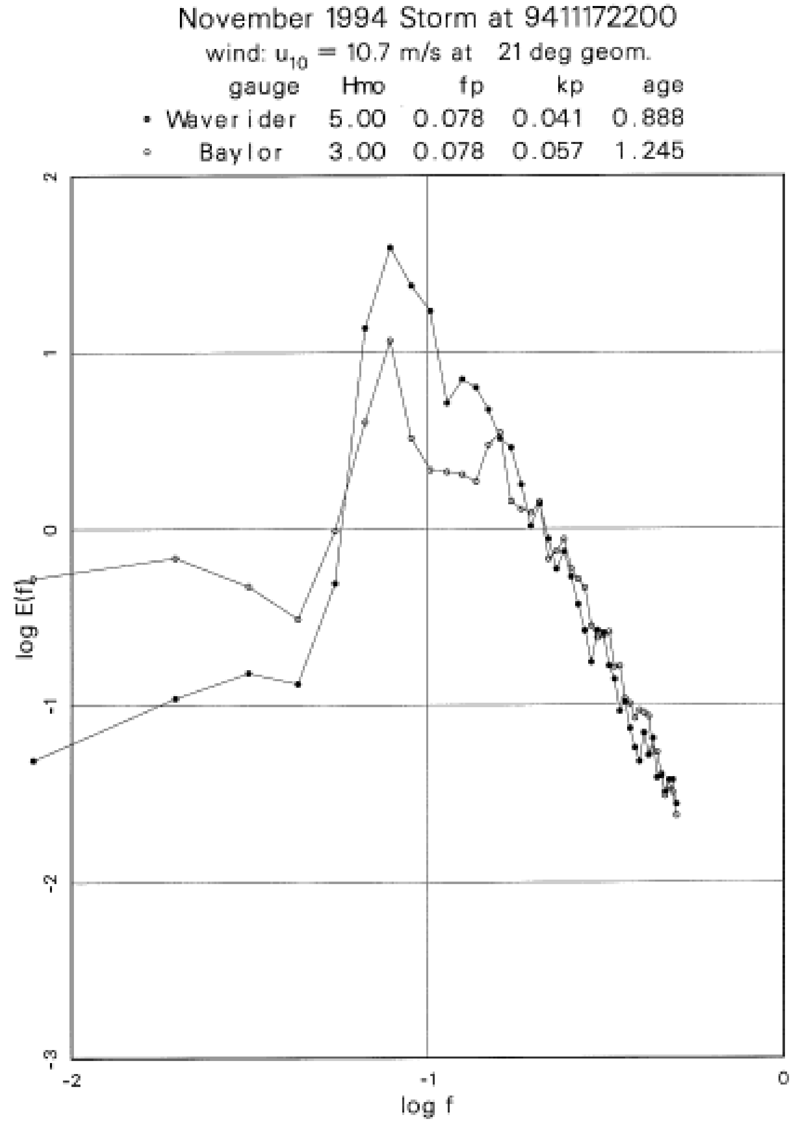
Some Implications

- Waves propagating into a coast

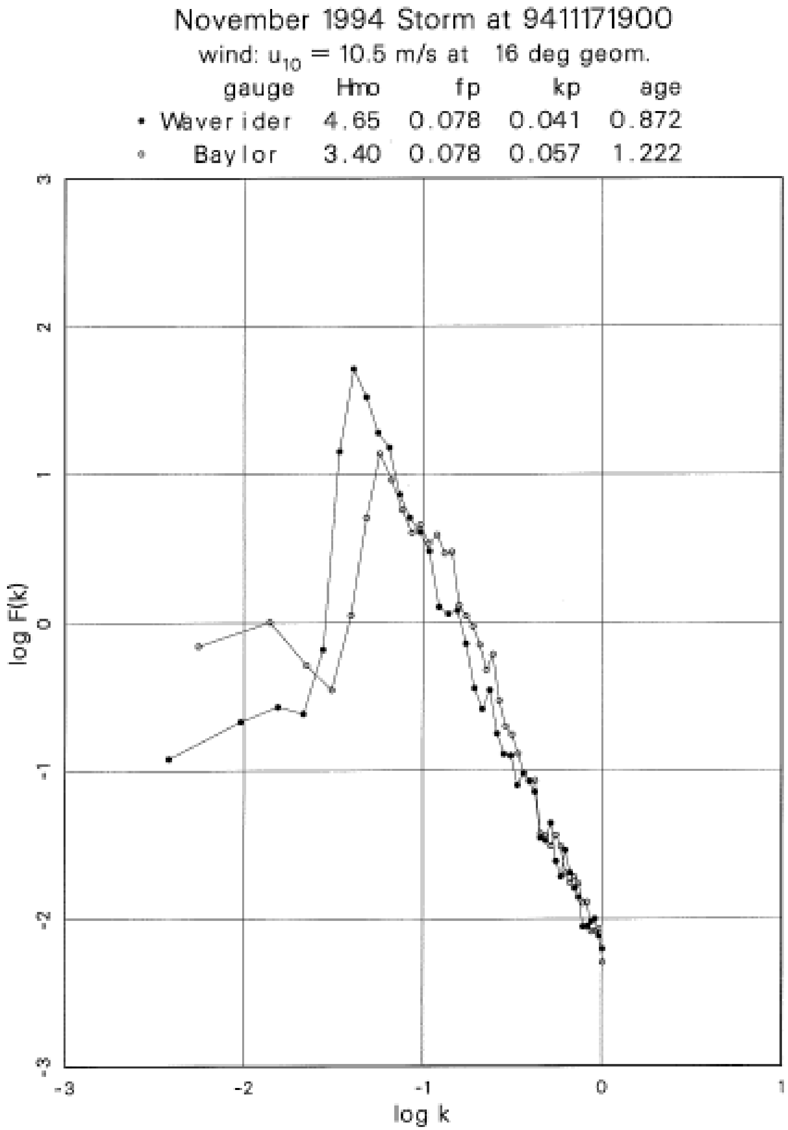
- Wave Set-up

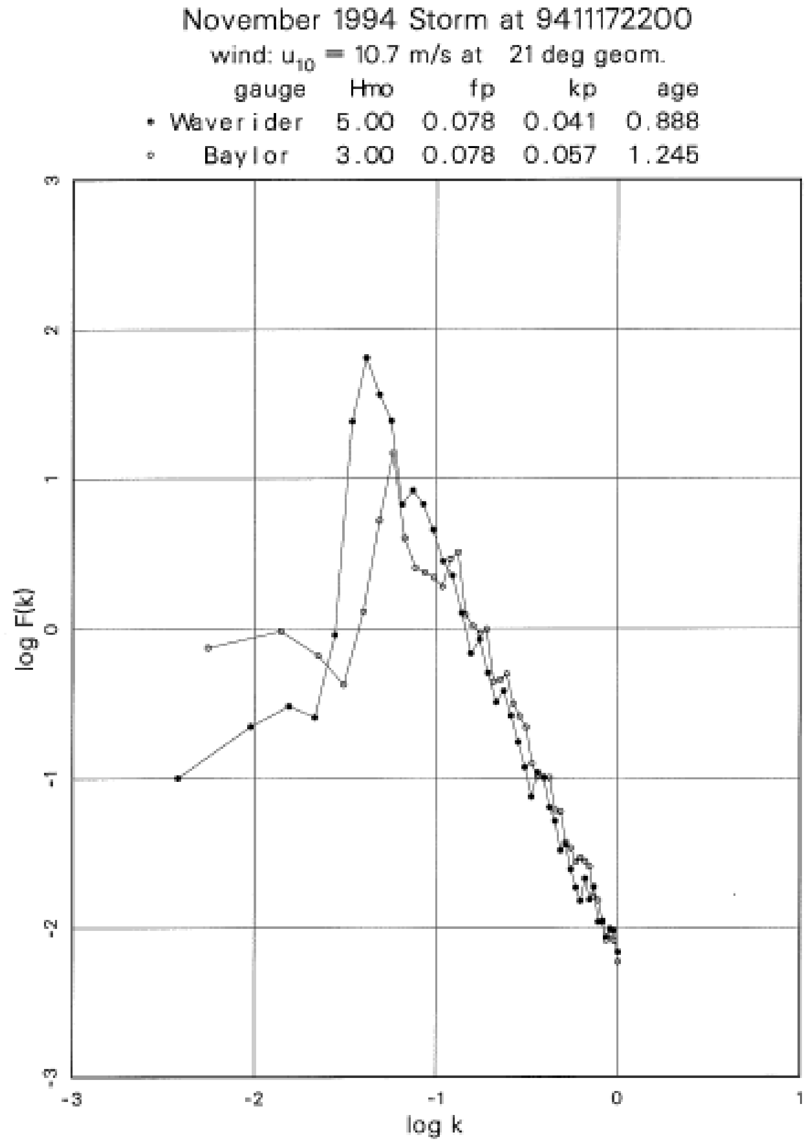
- Wave model source terms





Problem of transformation from waverider to Baylor gage viewed in frequency space. Dissipation at peak – suggests that we need a good source term in that region of the spectrum = breaking at the spectral peak??





Problem of transformation from waverider to Baylor gage viewed in wavenumber space. Wavenumber similarity (?) – suggests that we need a good source term in the high frequency region of the spectrum?? Note shift to k^{-3} form.

The contribution of the momentum flux into the water column to from the wave field, in the absence of winds, creates a (steady-state) slope which is dependent on the depth of water.

$$\frac{\partial \eta}{\partial y} \sim \frac{\Gamma_M}{h} \sim \frac{\Gamma_E}{hc}$$

where η is the water surface elevation,

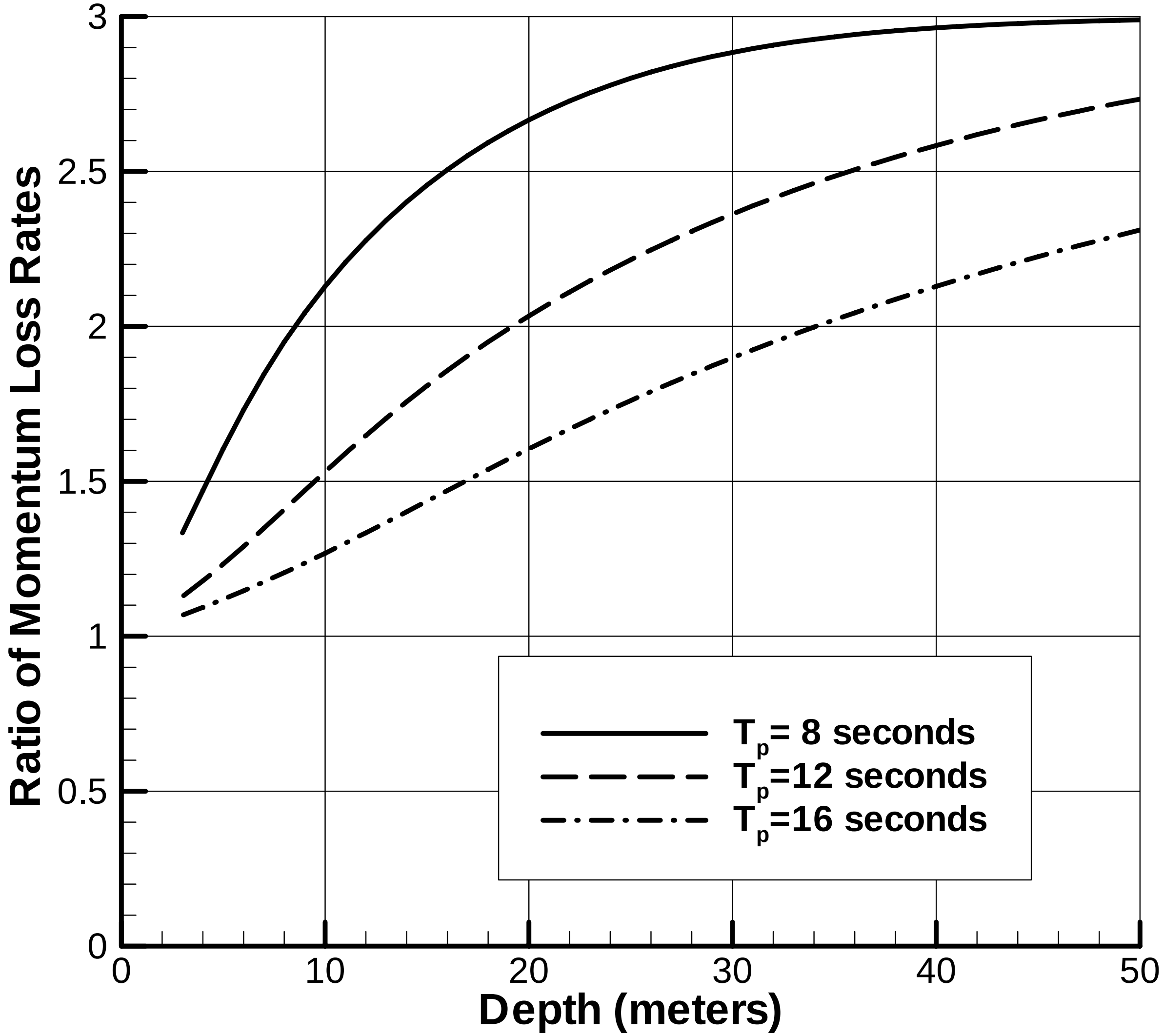
y is the direction normal to a straight coast,

Γ_{M} is the rate of flux of momentum from the wave field into the water column,

h is the water depth.

Thus, if wave breaking or other source terms removes energy in “deeper” water, the setup can be considerable reduced over the case of dissipation which removes energy in “shallower” water.

Effect of this breaking form on wave set-up could be very large.

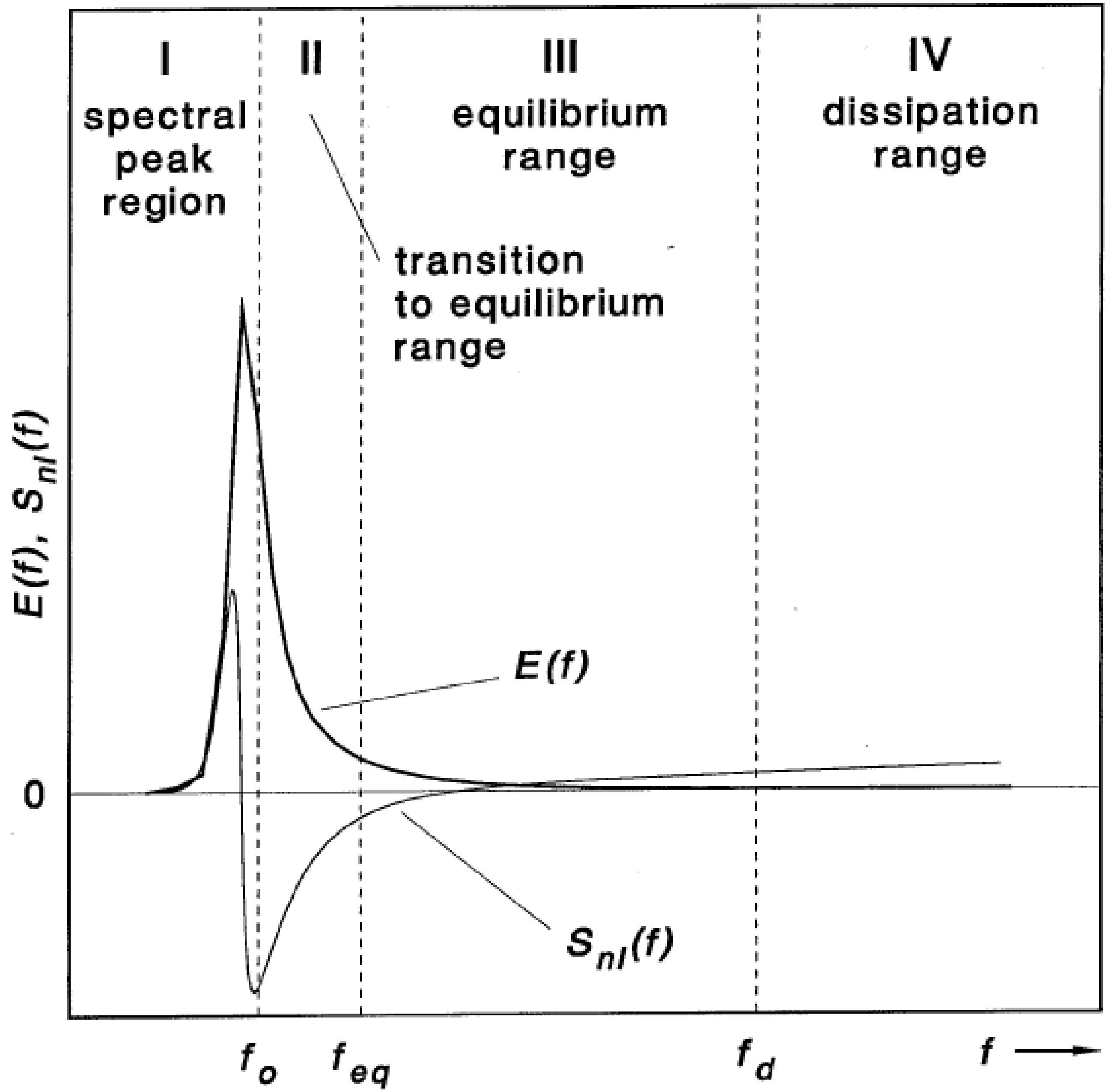


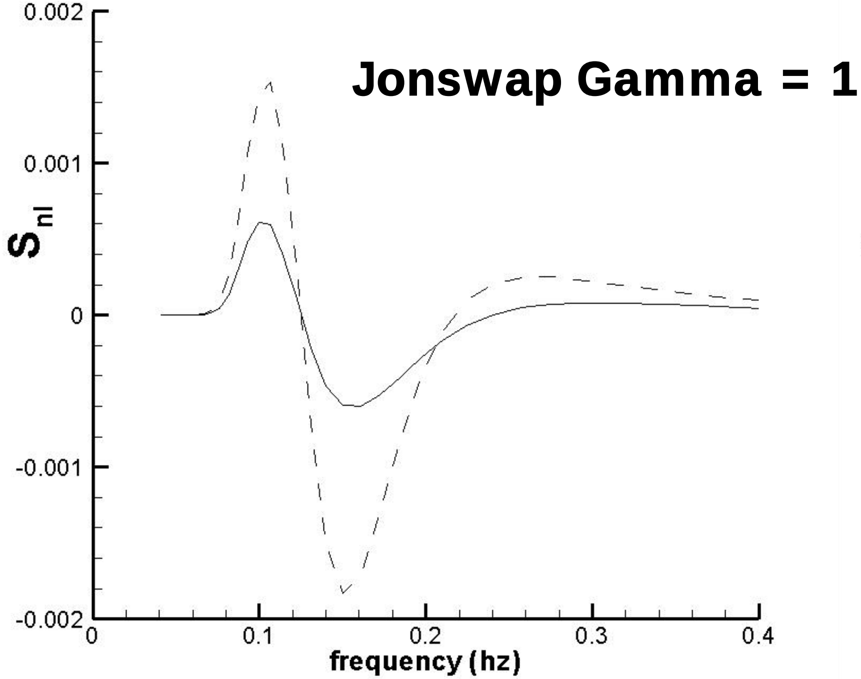
Calculated ratios of momentum lost from wave field given transition frequency equal to 3 time spectral peak

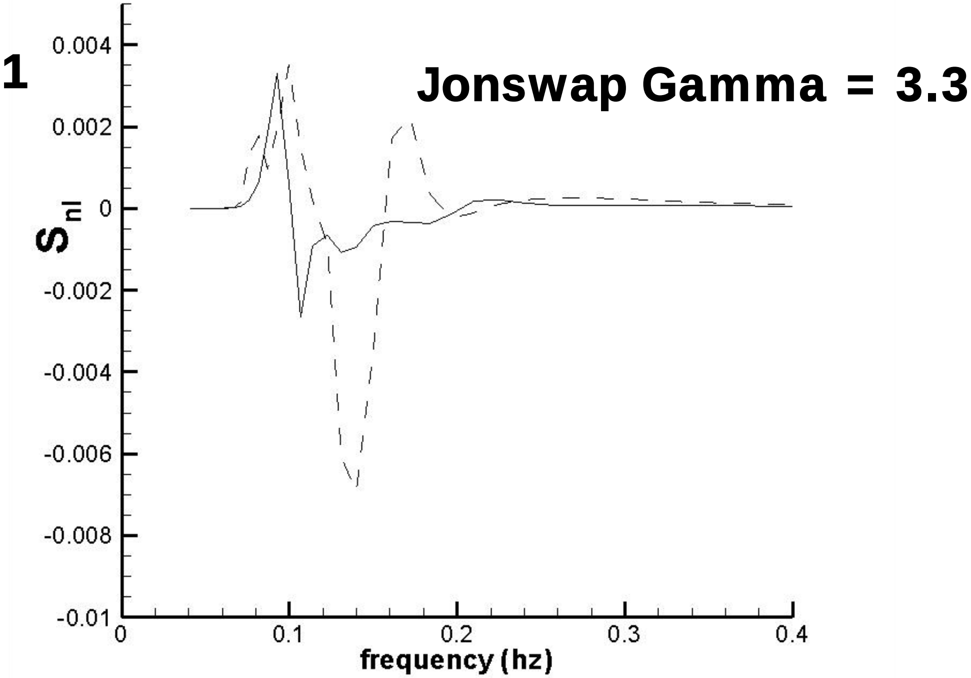
Possible new source term balance with improved Snl and high frequency breaking

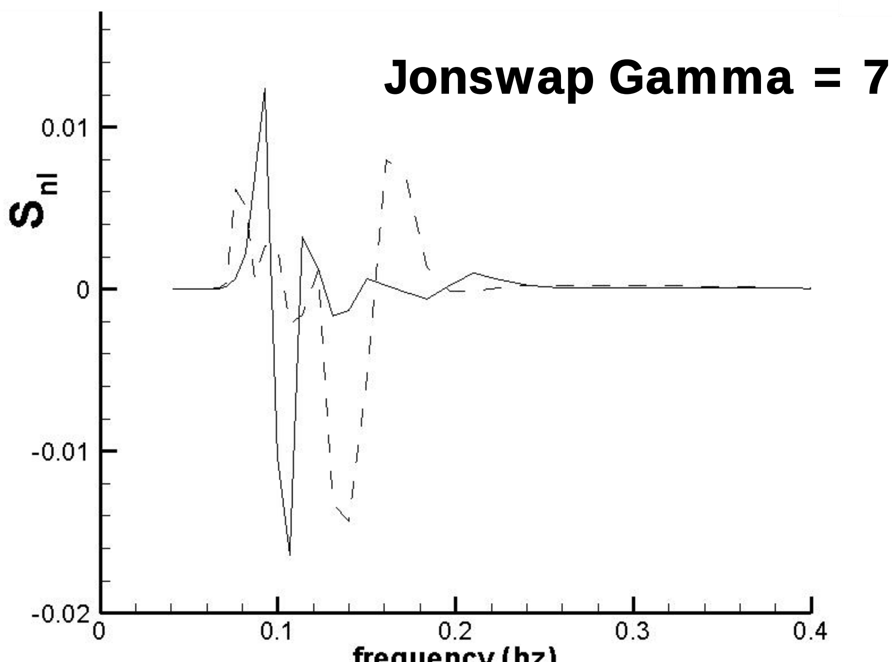
Major problem in existing models: DIA

Other source terms have to be tuned to compensate



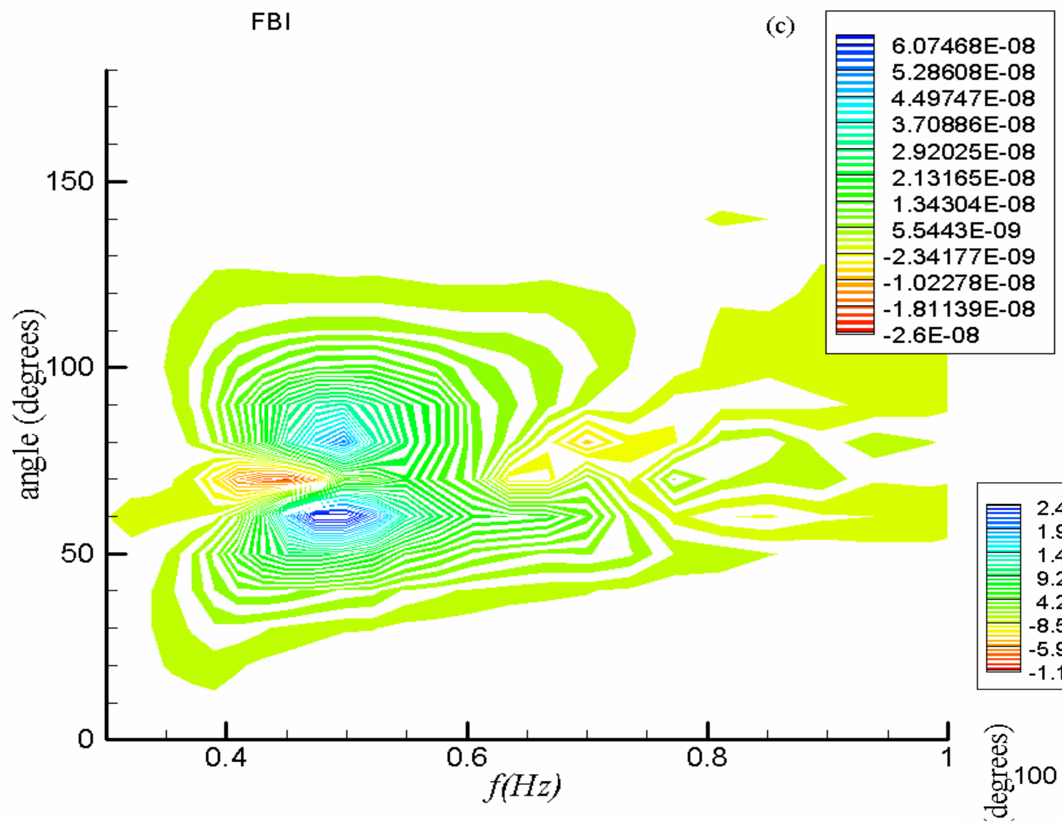






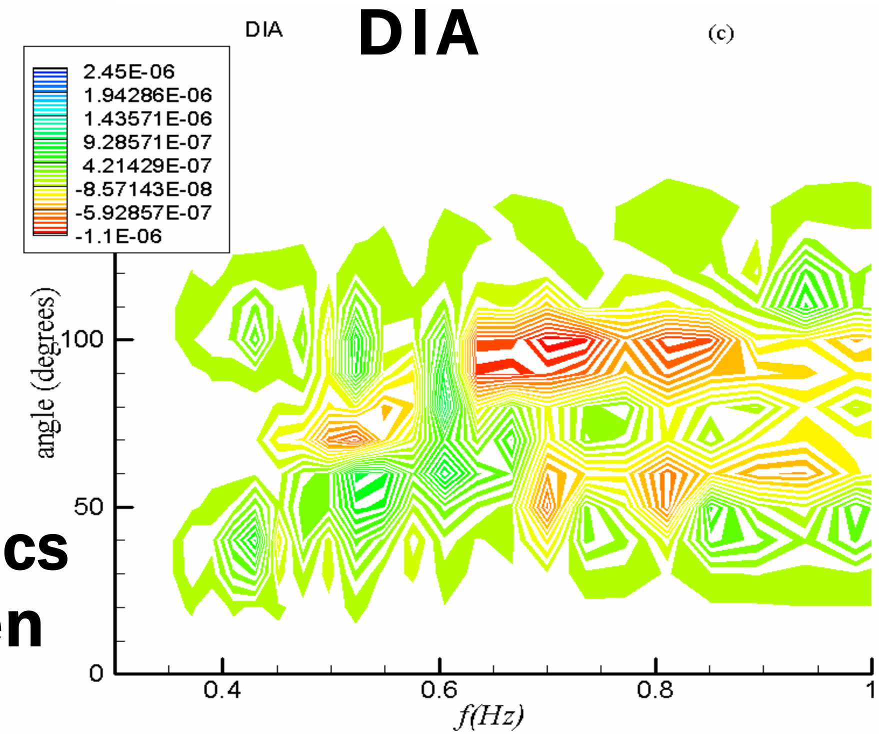
DIA does not give a consistent estimate of Snl for different peakednesses (i.e. it varies with wave age)

Full Boltzmann Integral



Comparison of DIA to WRT estimate of Snl for a measure spectrum

DIA



DIA directional characteristics become very “garbled” when directional characteristics are examined

WAY AHEAD: New Model

- TSA Replaces DIA
- High Frequency Breaking Replaces Distributed Breaking
- Solve for Wind Source for Closure

Questions?

QUESTIONS??