ON EXPERIMENTAL JUSTIFICATION OF WEAKLY TURBULENT NATURE OF GROWING WIND SEAS

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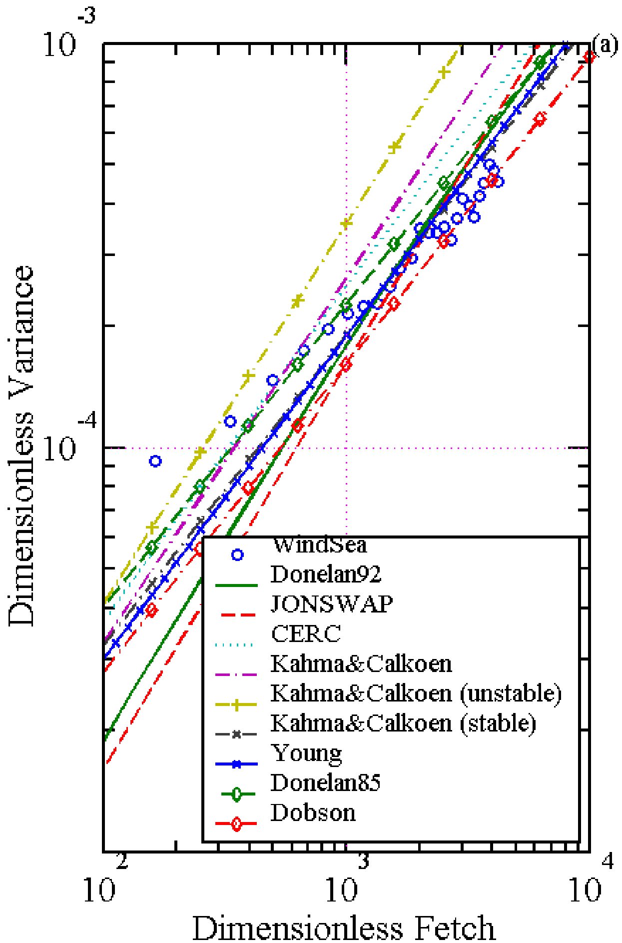
Power-law dependencies of fetch-limited wind-wave growth

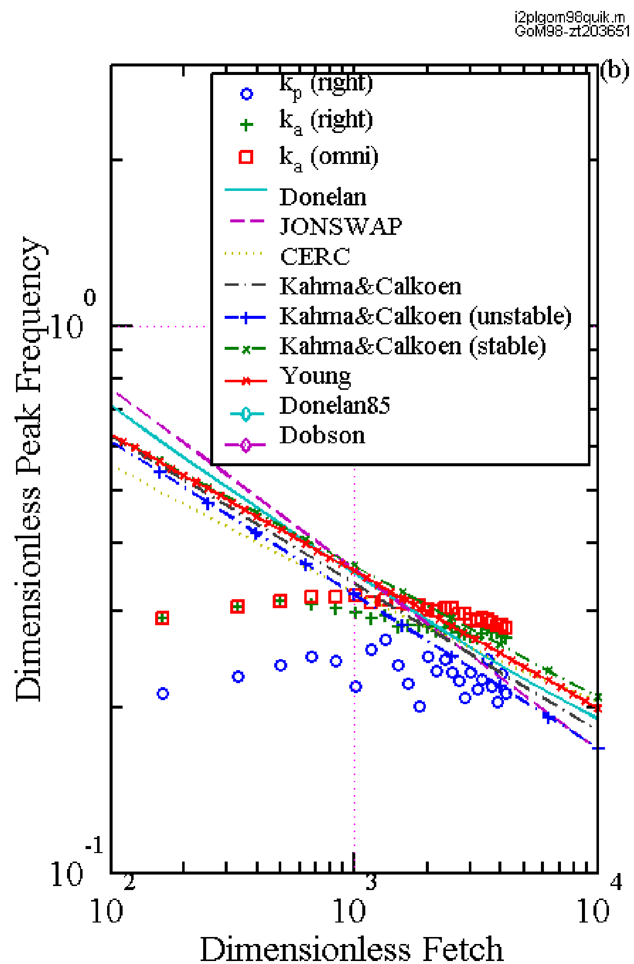
$$\tilde{\varepsilon} = \tilde{\varepsilon}_0 \chi^{p}$$

$$\tilde{\omega} = \tilde{\omega}_0 \chi^{-q}$$

$0.6 < p < 1.1$; $0.68 < 10^7 \varepsilon_0 < 18.6$;

$0.23 < q < 0.33$; $10.4 < \omega_0 < 22.6$





$\tilde{\varepsilon}_0, \tilde{\omega}_0, p, q$

fixed (universal)

$$\chi = xg / U_{10}^2;$$

$$\tilde{\varepsilon} = \varepsilon g^2 / U_{10}^4;$$

$$\tilde{\omega} = \omega_p U_{10} / g$$

Our thanks to Paul Hwang

St1. Perhaps it is time to abandon the idea that

a universal power law

for non-dimensional fetch-limited growth rate

is anything more than an idealization

Donelan, M., Skafel, M., Graber, H., Liu, P., Schwab, D. & Venkatesh, S., 1992, Atm.Ocean, 30(3)

St2. Perhaps it is time to accept the idea that

a universal power law

of weakly turbulent wave growth

is something more than an idealization

S. Badulin, A. Babanin, V. Zakharov, D. Resio, 2007, JFM, v.591

$$\varepsilon = \varepsilon_0 \chi^{p}$$

$$\omega_p = \omega_0 \chi^{-q}$$

$$\frac{\varepsilon \omega_p^4}{g^2} = \alpha_{ss} \left(\frac{{d\varepsilon}/{dt}\, \omega_p^3}{g^2} \right)^{1/3}$$

No wind speed scaling ! ε_0, ω_0, p, q are not fixed !
There is a family of power law dependencies governed by wave turbulence mechanisms

Methodology

Hypothesis: S_{nl} is a leading term of wind-wave balance

Theory: Asymptotic self-similar solutions for KinEq and generalization of Kolmogorov-Zakharov cascades in weak turbulence

Experiment: Weakly turbulent link of wave energy and net total input is consistent with more than 20 experimental dependencies of wave growth obtained for last 50 years

$$\frac{dn_k}{dt} = S_{nl} + S_{in} + S_{diss}$$ The Hasselmann equation

Just a hypothesis: nonlinearity dominates

$$S_{nl} >> S_{input}\,,\ \ S_{diss}$$

Split balance of wind-driven waves

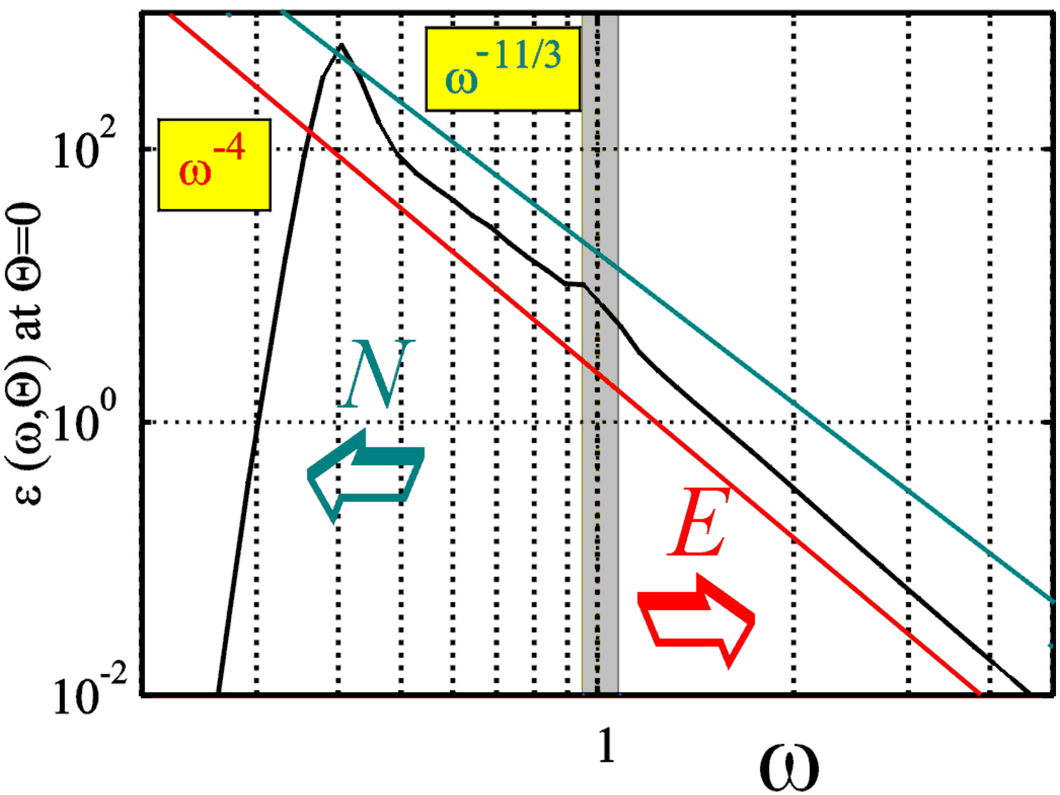
$$\frac{dn_k}{dt} = S_{nl}$$ Conservative KE

$$\left\langle \frac{dn_k}{dt} \right\rangle = \left\langle S_{in} + S_{diss} \right\rangle$$ Closure condition

Kolmogorov's cascades (Zakharov, PhD thesis 1966)

$$dn_k/dt = S_{nl}$$

Conservative KE is valid in "a transparency range" only (spectra tails *etc*)?



$$E^{(1)}(\omega,\theta) = C_p \frac{g^{4/3}P^{1/3}}{\omega^4}$$

Direct cascade (Zakharov & Filonenko 1966)

$$E^{(2)}(\omega,\theta) = C_q \frac{g^{4/3}Q^{1/3}}{\omega^{11/3}}$$

Inverse cascade (Zakharov & Zaslavskii 1983)

Power-law growth is described by non-homogeneous self-similar solutions of split balance model

$$\left\langle S_{in} + S_{diss} \right\rangle \sim x^{p-1}$$

$$\varepsilon = \varepsilon_0 x^{p}; \quad \omega_p = \omega_0 x^{-q}$$

Fetch-independent form leads to the Kolmogorov-Zakharov link of wave input and wave energy

$$\frac{\varepsilon \omega_p^4}{g^2} = \alpha_{ss} \left(\frac{d\varepsilon/dt\, \omega_p^3}{g^2} \right)^{1/3}$$

We do not need a notorious “transparency range”!

$$d\varepsilon/dt = const \quad \Rightarrow \quad \varepsilon \sim \omega^{-3} \quad \text{Toba's law}$$

Check this link for experimental power-law fits of non-dimensional energy and frequency

$$\chi = xg/U_{10}^2; \quad \tilde{\varepsilon} = \varepsilon g^2/U_{10}^4; \quad \tilde{\omega} = \omega_p U_{10}/g$$

$$\varepsilon_0 \omega_0^4 = \alpha_{ss}\left(\varepsilon_0 \omega_0^2/2\right)^{1/3} \chi^{-z};$$

$$z = \frac{10q - 2p - 1}{3}$$

$$p = \frac{10q - 1}{2}$$

Check directly (e.g. Zakharov 2005)

$$\alpha_{ss} = \left(\frac{2\varepsilon_0^2 \omega_0^{10}}{p}\right)^{1/3}$$

Estimate α_{ss}.

Att: Valid for constant *U* scaling!

Problems of our check

“…the effective fetch concept is a poor approximation…”

Kahma & Pettersson 1994, p.262

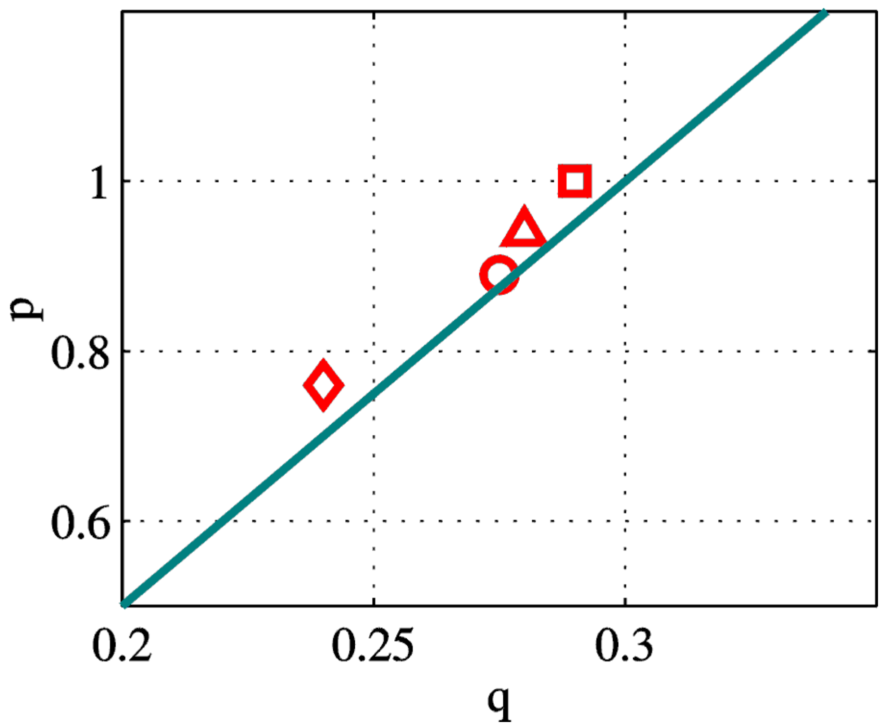
Data quality

1. **Wave tank data** – perfectly different physics
2. **“True” fetch** –
    1. time-to-fetch conversion
    2. wind-speed scaling – spurious correlations
3. **Composite data** – averaging of dependencies with different exponents and pre-exponents

Formal criteria to group dependencies in 4 groups ?

“Cleanest” dependencies

$$p = \frac{10q - 1}{2}$$



Black Sea

Babanin et al., 1996

US coast, N.Atlantic

Walsh et al 1989

Bothnian Sea, unstable

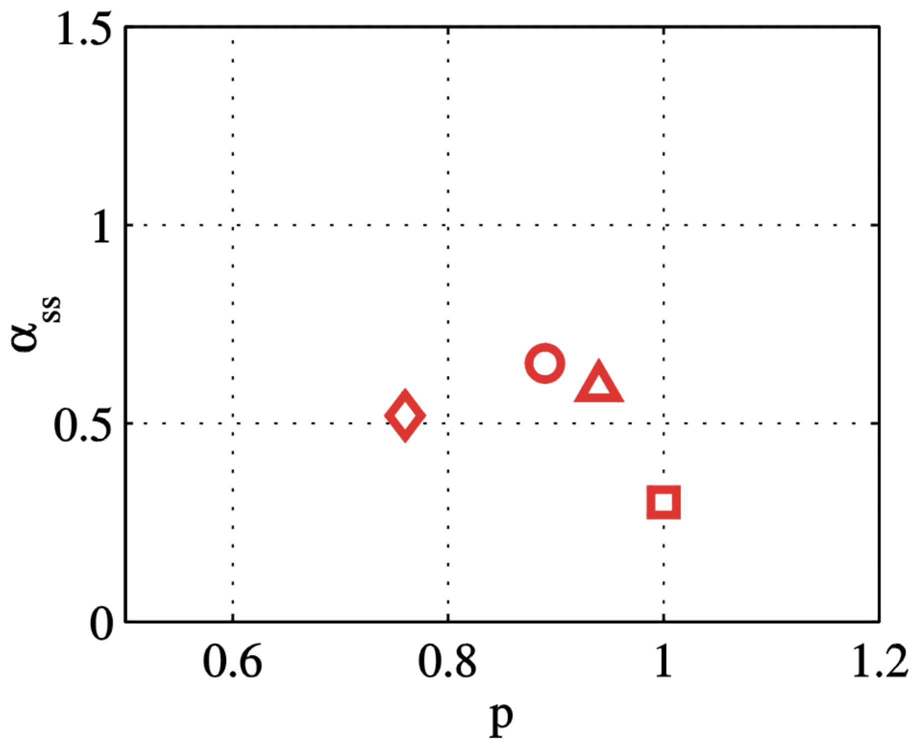
Kahma & Calkoen 1992

Bothnian Sea, stable

Kahma & Calkoen 1992

"Cleanest" dependencies

$$\varepsilon_0 \omega_0^4 = \alpha_{ss} \left(0.5 p \varepsilon_0 \omega_0^2\right)^{1/3}$$



Black Sea
Babanin et al., 1996

US coast
Walsh et al 1989

Bothnian Sea, unstable
Kahma & Calkoen 1992

Bothnian Sea, stable
Kahma & Calkoen 1992

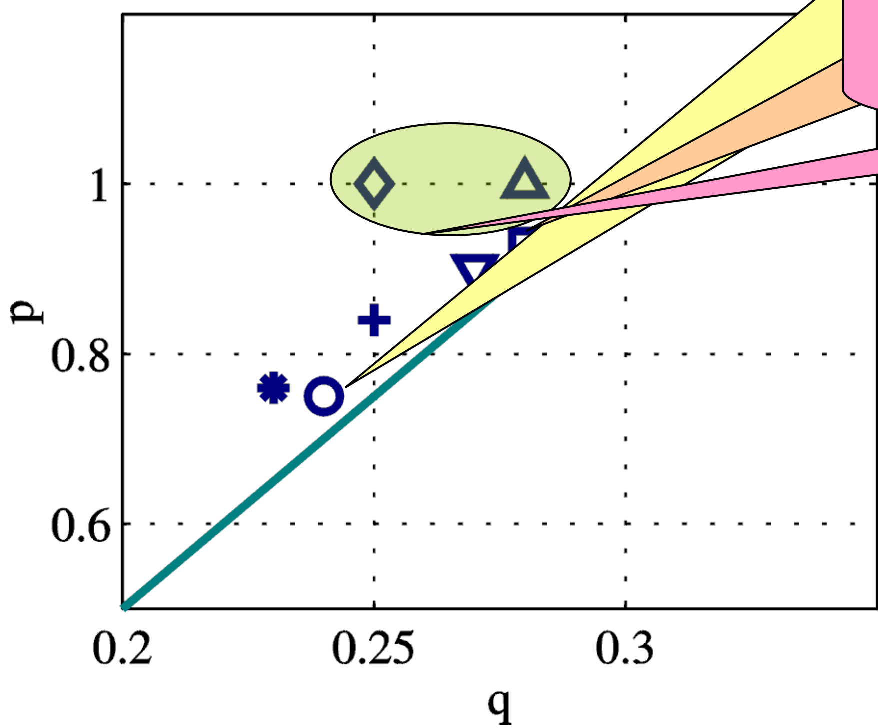
Composite data

$$p = \frac{10q - 1}{2}$$

“the fetch-averaged wind must represent a time and space history of the wave field”

“the effective fetch concept is a poor approximation”

“many of the spectra from the JONSWAP experiment show more structure”
Kahma & Calkoen (1992)



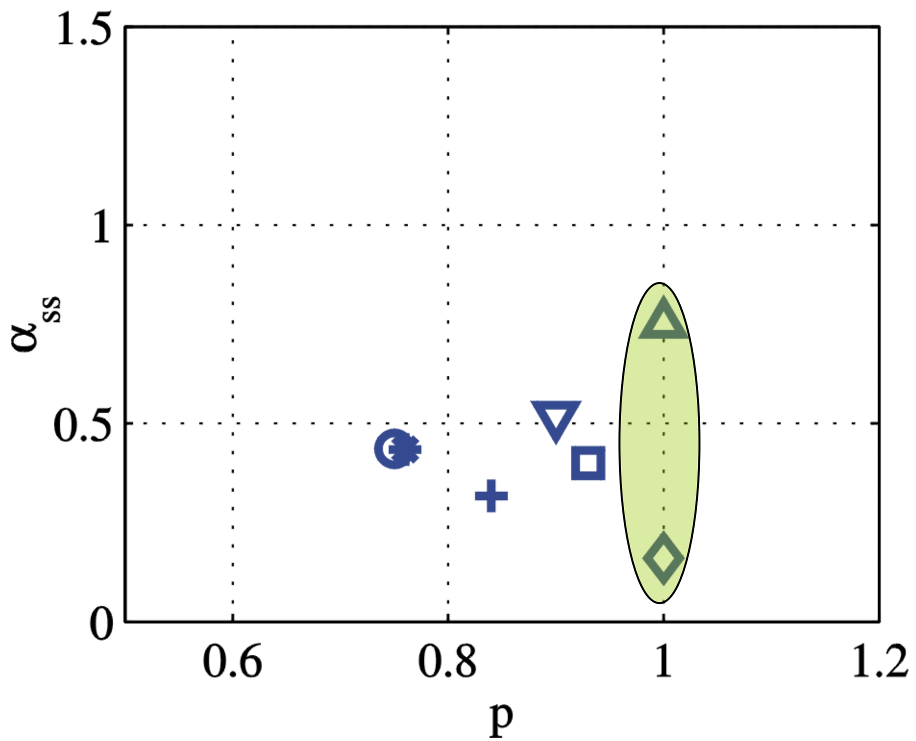
Kahma & Calkoen 1992, composite

Lake Ontario *Donelan et al.1985*

CERC (1977) by Young (1999)

Composite data

$$\varepsilon_0 \omega_0^4 = \alpha_{ss} \left(0.5 p \varepsilon_0 \omega_0^2\right)^{1/3}$$



Dobson et al. 1989

Kahma & Pettersson 1994

JONSWAP by *Davidan 1980*

JONSWAP by *Phillips 1977*

Kahma & Calkoen 1992, composite

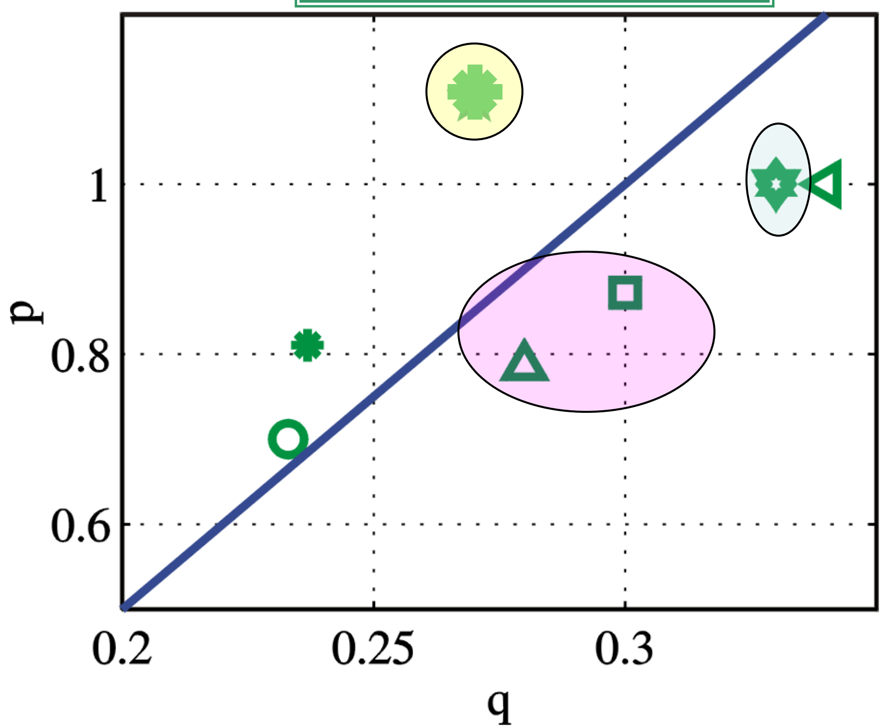
Lake Ontario
Donelan et al.1985

CERC (1977) by Young (1999)

“Bad” dependencies

(one-point measurements, time-to-fetch conversion, pre-determined exponents etc.)

$$p = \frac{10q - 1}{2}$$



Wen et al. 1989

Evans & Kibblewhite 1990, neutral

Evans & Kibblewhite 1990, stable

***Kahma & Calkoen 1981,86*, rapid**

Kahma & Calkoen 1981, average

Donelan et al.1992

Hwang & Wang (2004, 2006)

Ross 1978, *Atlantic*, stable

Liu & Ross 1980, *L.Michigan*, unstable

Davidan 1996, u* scaling

“Bad” dependencies

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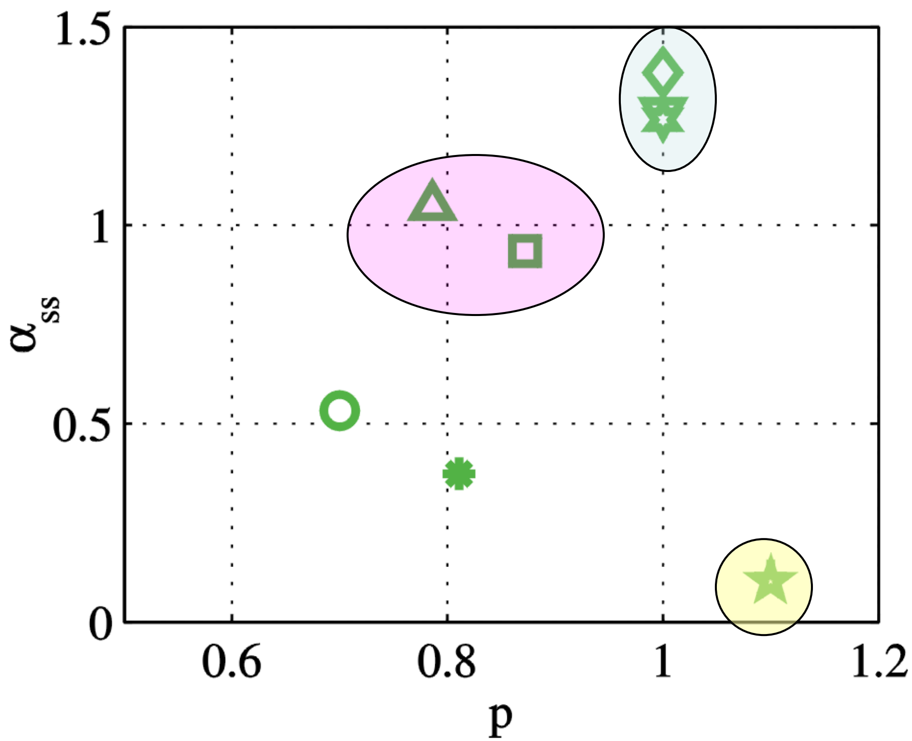
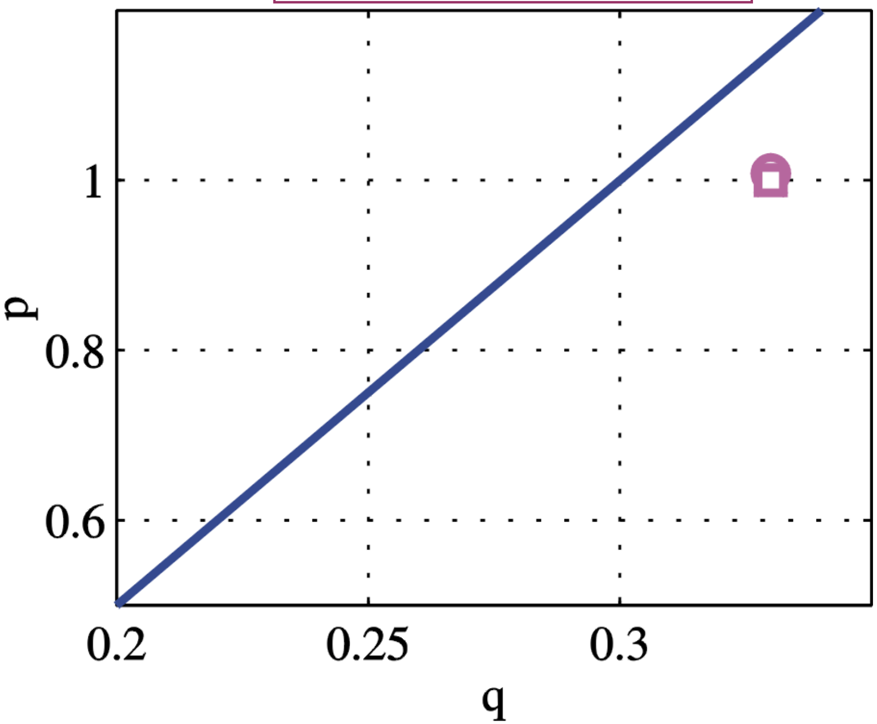
$$\varepsilon_0 \omega_0^4 = \alpha_{ss} \left(0.5 p \varepsilon_0 \omega_0^2\right)^{1/3}$$



- *Wen et al. 1989*
- *Evans & Kibblewhite 1990*, neutral
- *Evans & Kibblewhite 1990*, stable
- ***Kahma & Calkoen 1981,86*, rapid**
- *Kahma & Calkoen 1981*, average
- ***Donelan et al.1992***
- *Hwang & Wang (2004, 2006)*
- *Ross 1978, Atlantic*, stable
- *Liu & Ross 1980, L.Michigan*, unstable
- *Davidan 1996*, u* scaling (out of scale)

"Sea + wave tank" dependencies

$$p = \frac{10q - 1}{2}$$

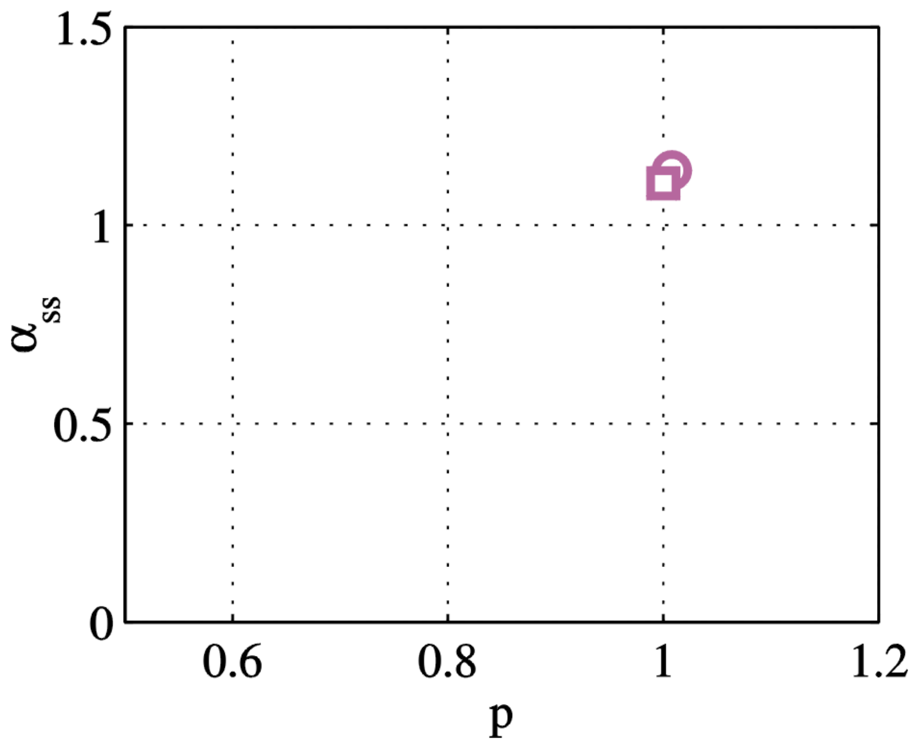


JONSWAP,
Hasselmann et al. 1973

Mitsuyasu et al. 1971

“Sea+laboratory” dependencies

$$\varepsilon_0 \omega_0^4 = \alpha_{ss} \left(0.5 p \varepsilon_0 \omega_0^2\right)^{1/3}$$



JONSWAP,
Hasselmann et al. 1973

Mitsuyasu et al. 1971

Concluding remarks

- Wave growth dependencies for ε and ω_p are universal in the sense of weak turbulence law, they are governed by a rigid link of total energy and total net wave input. One-half of available experimental dependencies are consistent with weakly turbulent scenario of wave growth

- Basic parameter of weakly turbulent wave growth α_{ss} is estimated for the first time

Linguistic aspect

Flexible=Fluxible

Thank you for attention

Special thanks to *Oceanweather Inc.*

Summary

- Correspondence to weakly turbulent wave growth law for more than 20 fetch-limited dependencies is analyzed;

$$\varepsilon = \varepsilon_0 \chi^{p}; \quad \omega_p = \omega_0 \chi^{-q}$$

- Self-similarity parameter α_{ss} is estimated

$$\frac{\varepsilon \omega_p^4}{g^2} = \alpha_{ss} \left(\frac{d\varepsilon/dt\, \omega_p^3}{g^2} \right)^{1/3}$$

Motivation I

$$\tilde{\varepsilon} = \tilde{\varepsilon}_0 \chi^{p}$$

$$\tilde{\omega} = \tilde{\omega}_0 \chi^{-q}$$

$$\chi = xg / U_{10}^{2}; \quad \tilde{\varepsilon} = \varepsilon g^{2} / U_{10}^{4}; \quad \tilde{\omega} = \omega_p U_{10} / g$$

$\tilde{\varepsilon}_0, \tilde{\omega}_0$ – fixed (universal)

Perhaps it is time to abandon the idea that a universal power law for non-dimensional fetch-limited growth rate is anything more than an idealization

Donelan, M., Skafel, M., Graber, H., Liu, P., Schwab, D. & Venkatesh, S., 1992, Atm.Ocean, 30(3)