Comparison of Hasselmann and Dynamical Equations

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Dynamical equations :

$$\eta_t = \hat{k}\psi - (\nabla(\eta\nabla\psi)) - \hat{k}[\eta\hat{k}\psi] + \hat{k}(\eta\hat{k}[\eta\hat{k}\psi]) + \tfrac{1}{2}\Delta[\eta^2\hat{k}\psi] + \tfrac{1}{2}\hat{k}[\eta^2\Delta\psi] + \hat{\gamma}\eta$$

$$\psi_t = -g\eta - \tfrac{1}{2}[(\nabla\psi)^2 - (\hat{k}\psi)^2] - [\hat{k}\psi]\hat{k}[\eta\hat{k}\psi] - [\eta\hat{k}\psi]\Delta\psi + \hat{\gamma}\eta$$

$$\hat{k}\psi = \tfrac{1}{2\pi}\int k\psi_k e^{-ikr}\,dk_x dk_y$$

Hasselmann (kinetic) equation :

$$\frac{\partial n}{\partial t} = \int |T_{k123}|^2 \left(n_2 n_3 (n_1 + n_k) - n_1 n_k (n_2 - n_3)\right)\delta(k + k_1 - k_2 - k_3)\delta(\omega_k + \omega_1 - \omega_2 - \omega_3)dk_1 dk_2 dk_3 + \gamma_k n_k$$

Two reasons why the weak turbulent theory could fail:

1. ***Coherent events - solitons, quasi-solitons, wave collapses or wave-breakings***

2. ***Finite size of the system – discrete Fourier space:***

Quazi-resonances:

$$\omega_1 + \omega_2 = \omega_3 + \omega_4 + \delta$$

$$k_1 + k_2 = k_3 + k_4$$

Dynamic equations:

$2\pi \times 2\pi$ domain of 4096x512 point in real space

Hasselmann equation:

domain of 71x36 points in frequency-angle space

Three damping terms:

1. Hyper-viscous damping $\gamma_k = C(k-1024)^2$

2. WAM cycle 3 white-capping damping

3. WAM cycle 4 white-capping damping

4. New damping term

WAM Dissipation Function:

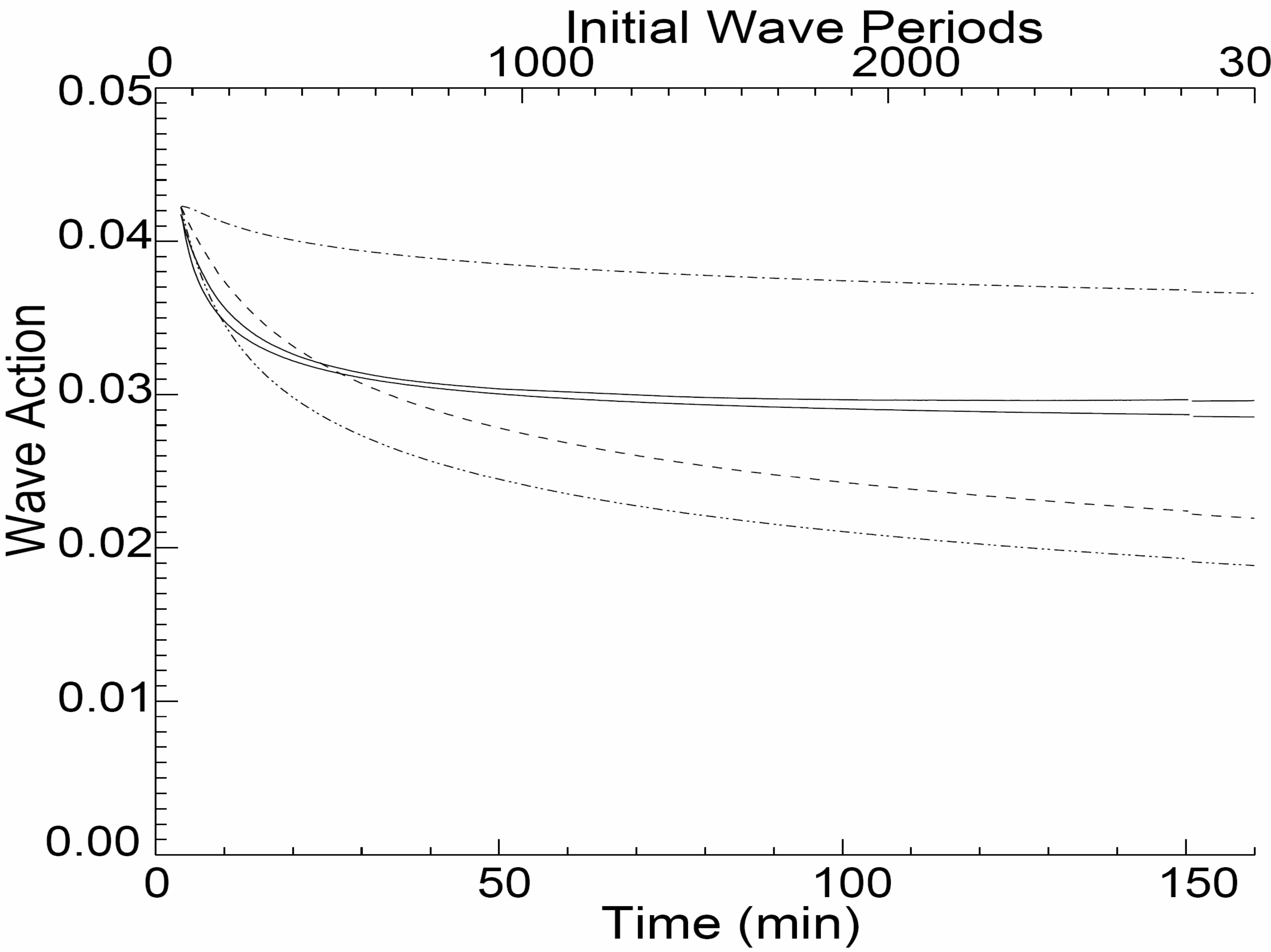
$$S_{ds}(\omega,\theta) = -C_{ds}\left(1-\sigma+\sigma\frac{k}{\tilde{k}}\right)\left(\frac{\tilde{S}}{\tilde{S}_{PM}}\right)^4 \tilde{\omega}\frac{k}{\tilde{k}}E(\omega,\theta)$$

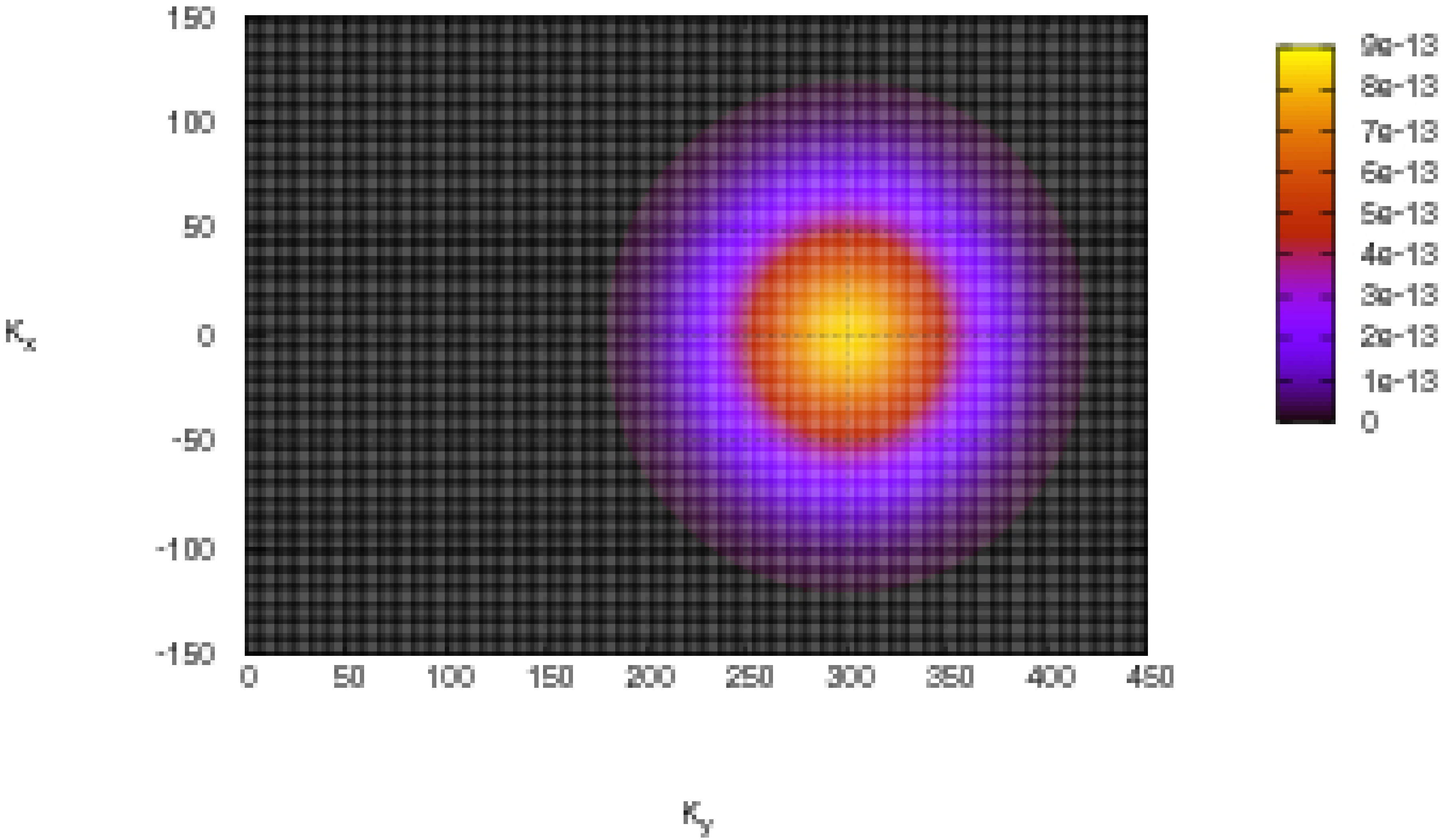
$$\tilde{S} = \tilde{k}\sqrt{E_{tot}}$$

$$\tilde{S}_{PM} = (3.02\cdot 10^{-3})^{1/2}$$

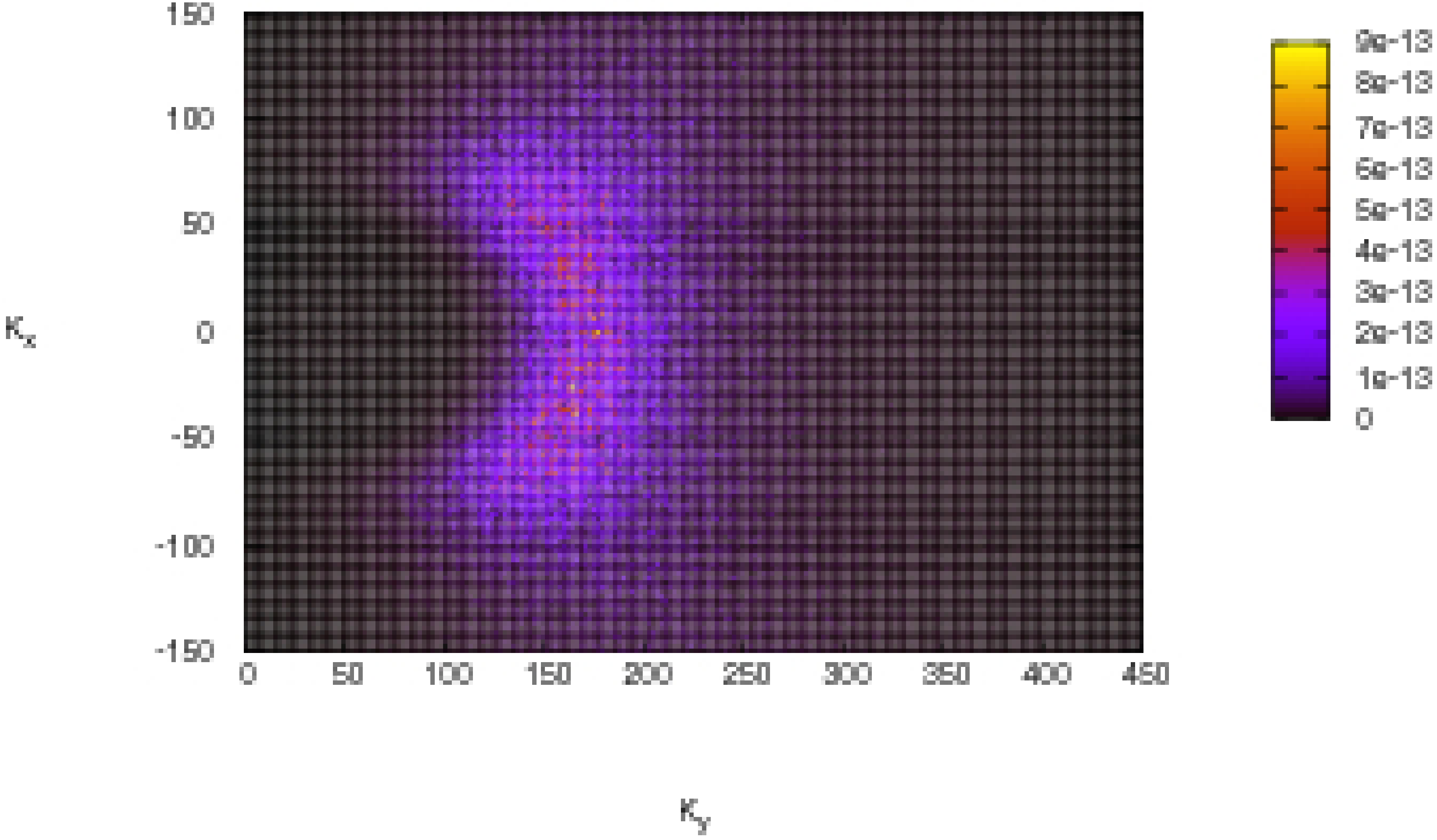
WAM cycle 3: $C_{ds} = 2.36\times 10^{-5}, \delta = 0.5$ *Komen 1984*

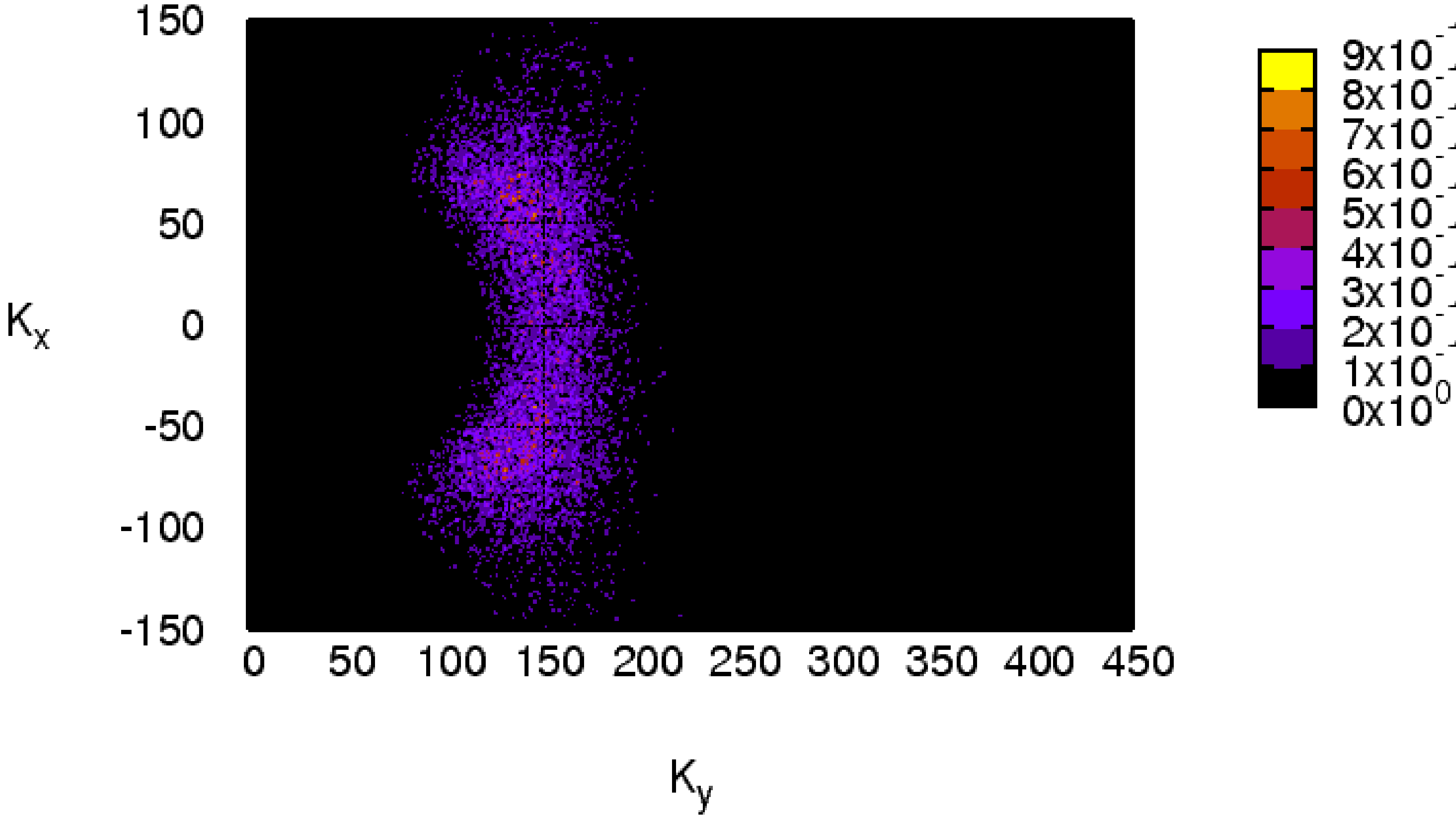
WAM cycle 4: $C_{ds} = 4.10\times 10^{-5}, \quad \delta = 0.0$ *Janssen 1992*
Gunter 1992
Komen 1994


Initial Wave Periods
0
1000
2000
30
0.05
0.04
0.03
0.02
0.01
0.00
Wave Action
0
50
100
150
Time (min)

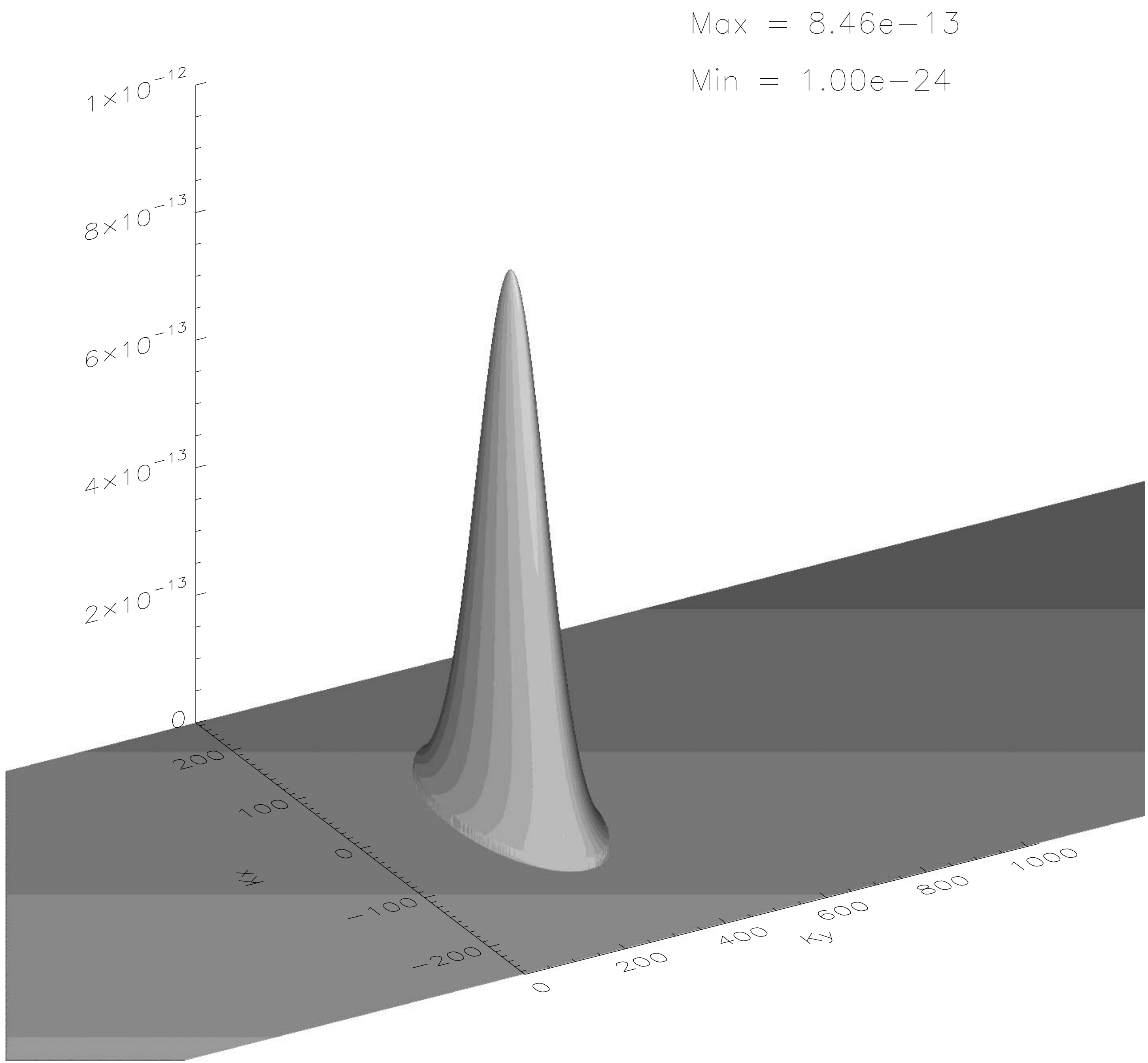


150
100
50
0
-50
-100
-150
K_x
0
50
100
150
200
250
300
350
400
450
K_y
9e-13
8e-13
7e-13
6e-13
5e-13
4e-13
3e-13
2e-13
1e-13
0

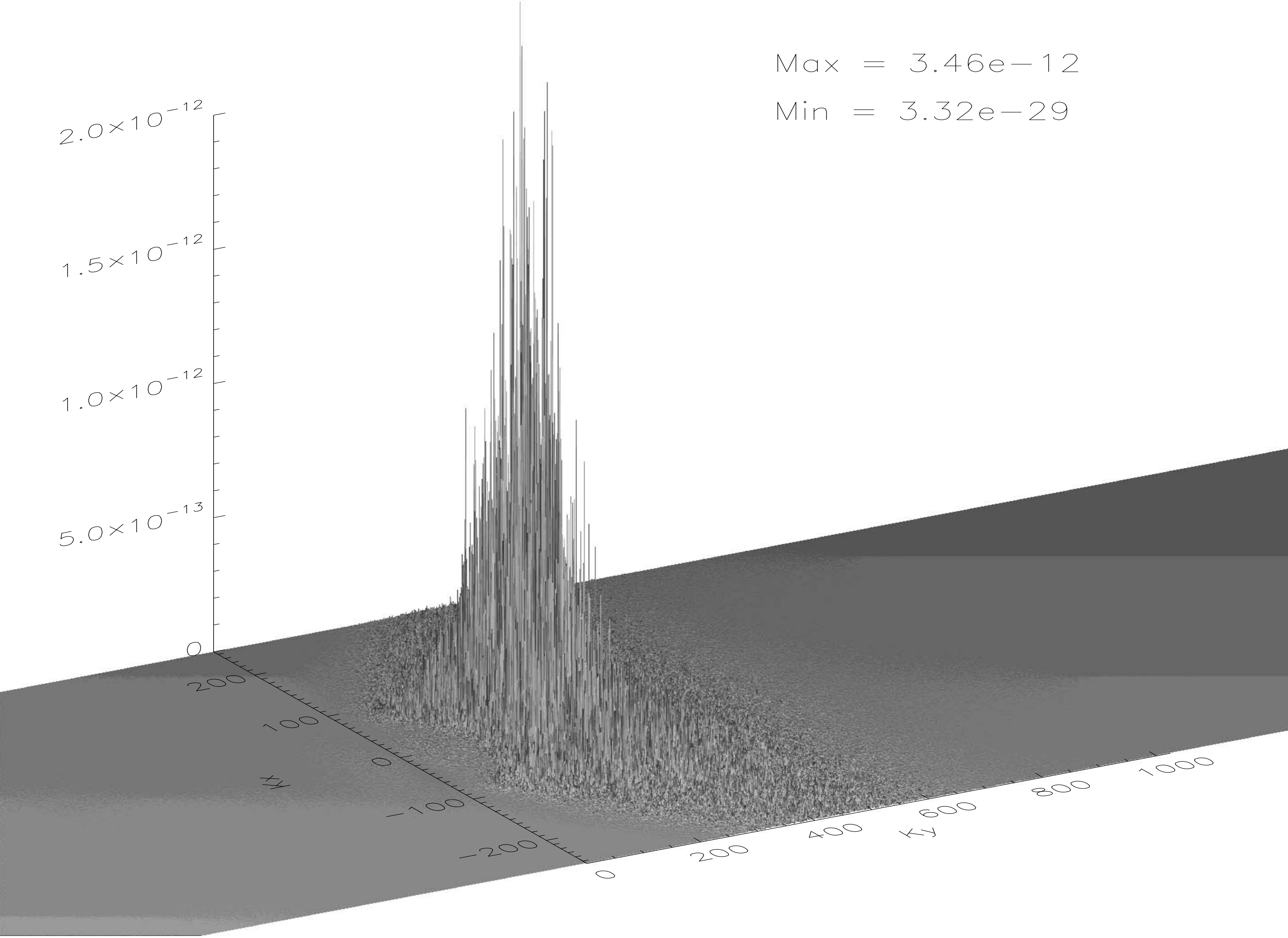

150
100
50
0
-50
-100
-150
K_x
0
50
100
150
200
250
300
350
400
450
K_y
9e-13
8e-13
7e-13
6e-13
5e-13
4e-13
3e-13
2e-13
1e-13
0



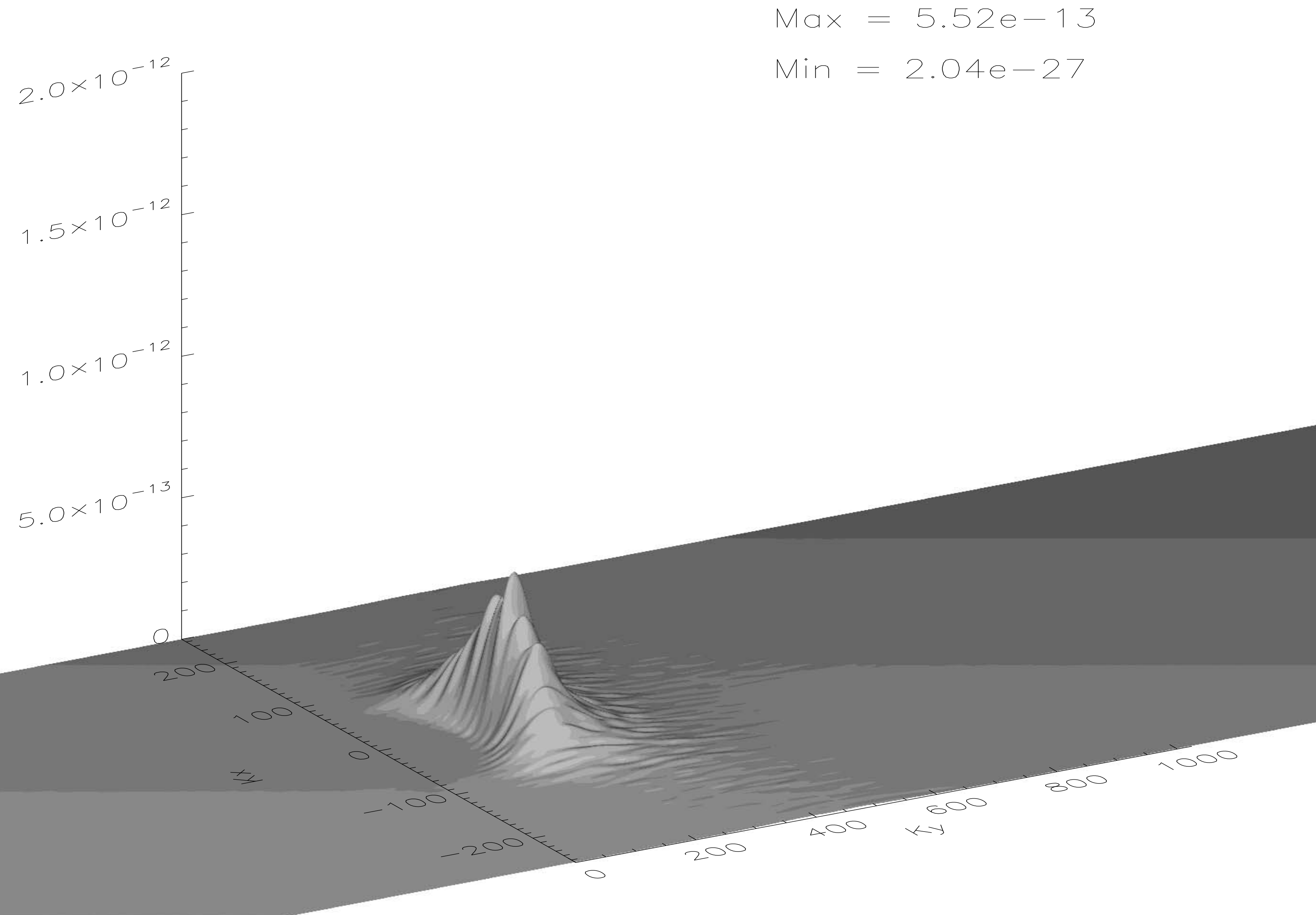
150
100
50
0
-50
-100
-150
K_x
0
50
100
150
200
250
300
350
400
450
K_y
9x10
8x10
7x10
6x10
5x10
4x10
3x10
2x10
1x10
0x10^0

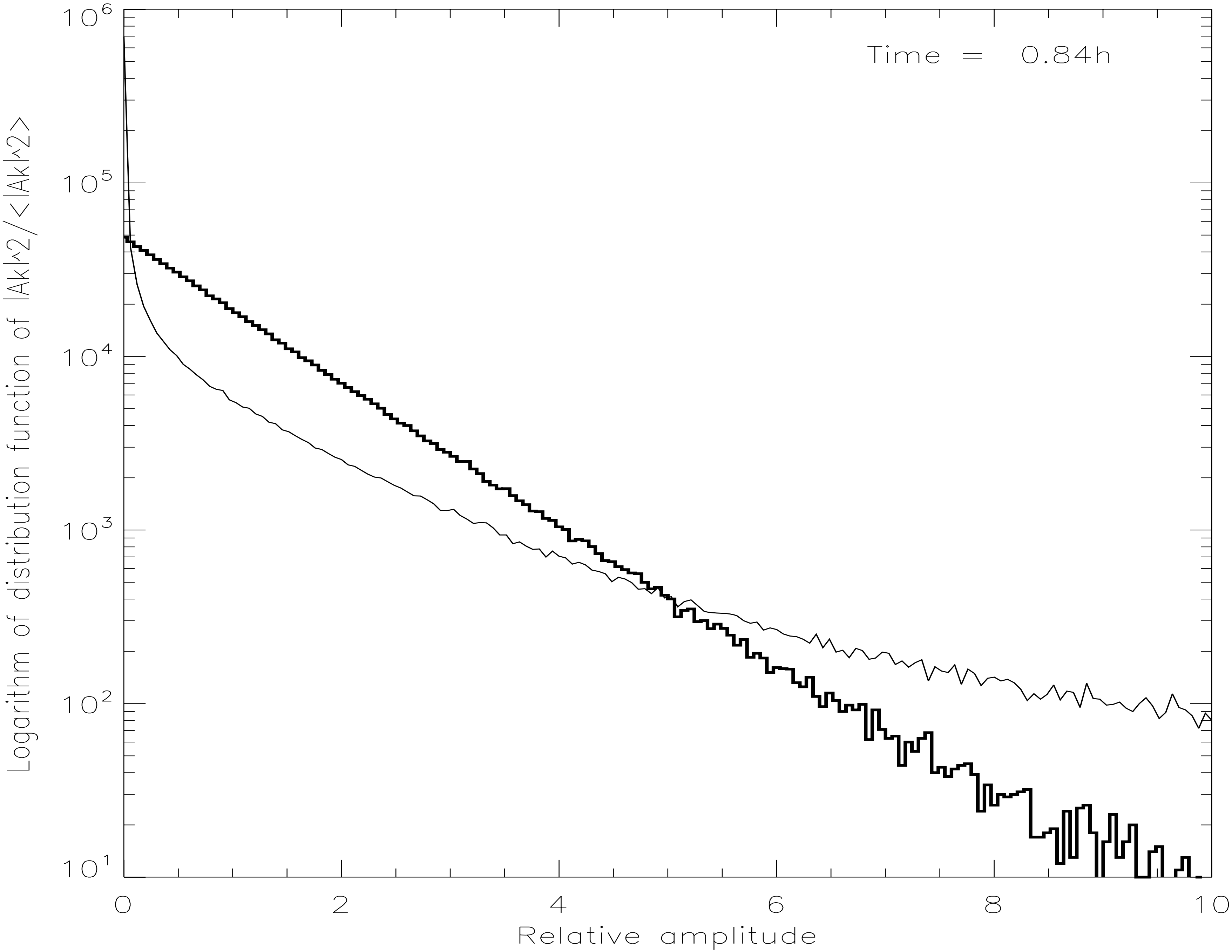


Max = 8.46e−13
Min = 1.00e−24
1×10^{-12}
8×10^{-13}
6×10^{-13}
4×10^{-13}
2×10^{-13}
0
200
100
0
Kx
−100
−200
0
200
400
Ky
600
800
1000

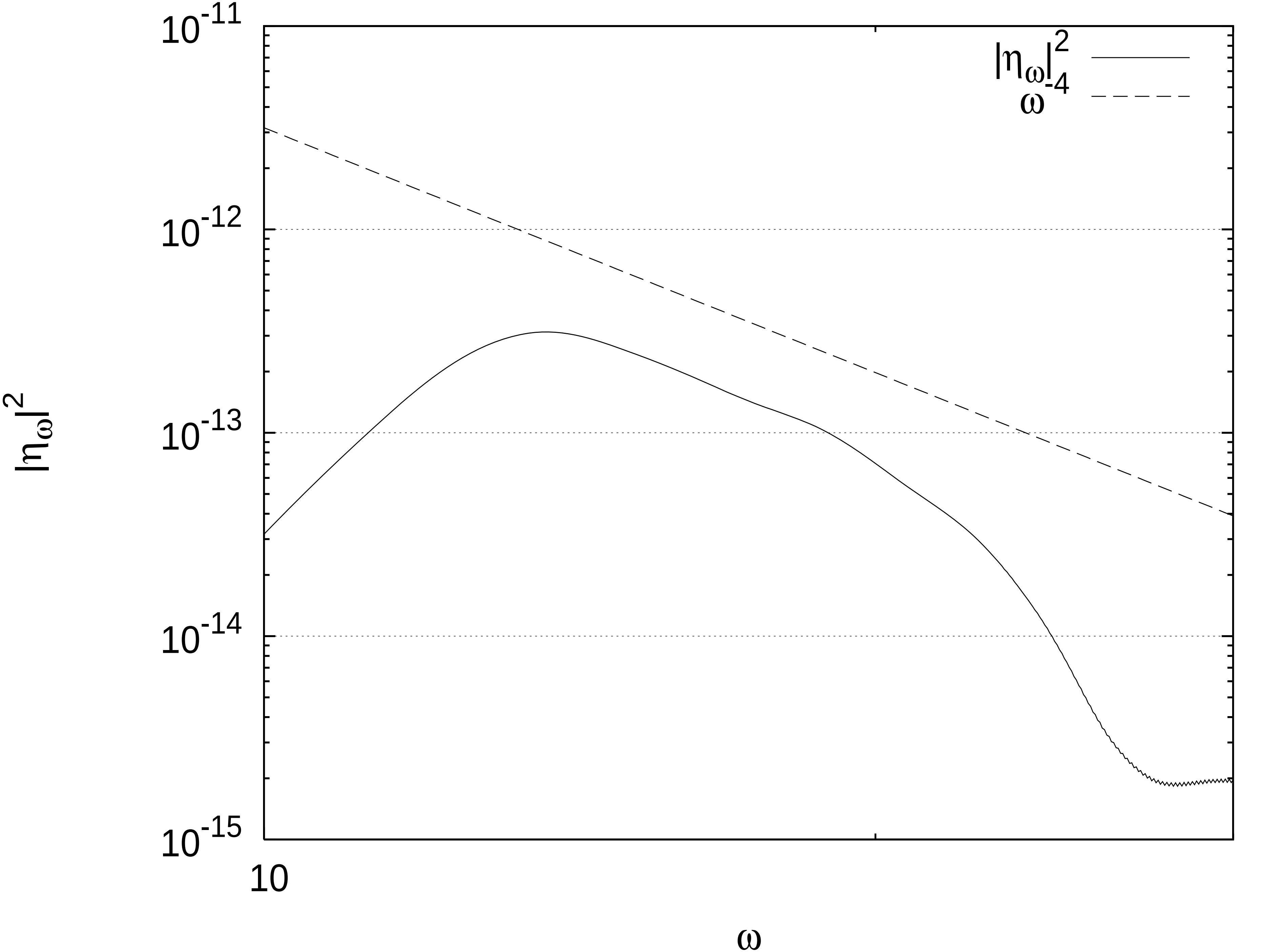


Max = 3.46e−12
Min = 3.32e−29
2.0×10−12
1.5×10−12
1.0×10−12
5.0×10−13
0
200
100
0
−100
−200
Kx
0
200
400
600
800
1000
Ky

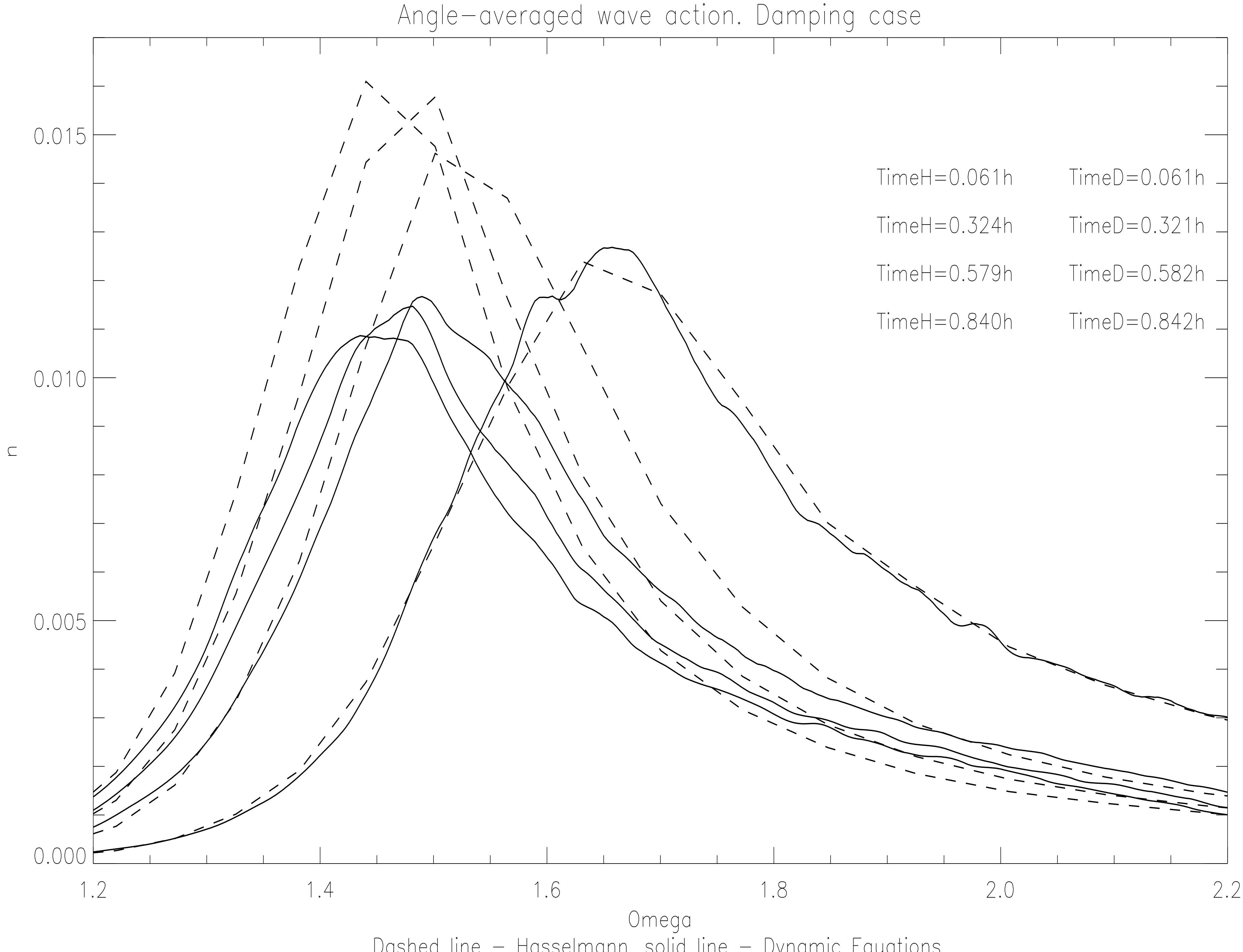

Max = 5.52e−13
Min = 2.04e−27
2.0×10−12
1.5×10−12
1.0×10−12
5.0×10−13
0
200
100
0
−100
−200
Kx
0
200
400
600
800
1000
Ky



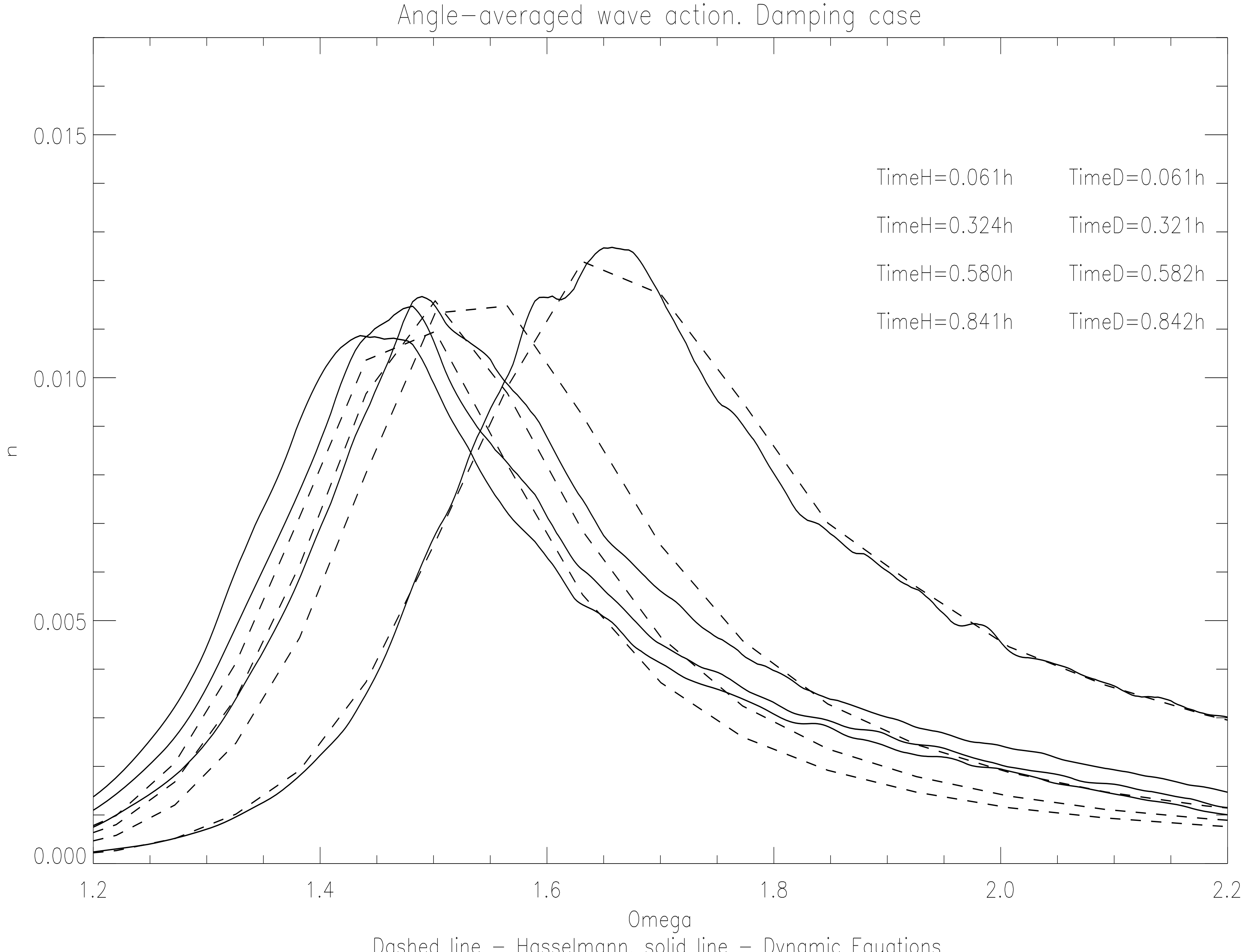
Time = 0.84h
Logarithm of distribution function of |Ak|^2/<|Ak|^2>
Relative amplitude
10^6
10^5
10^4
10^3
10^2
10^1
0
2
4
6
8
10



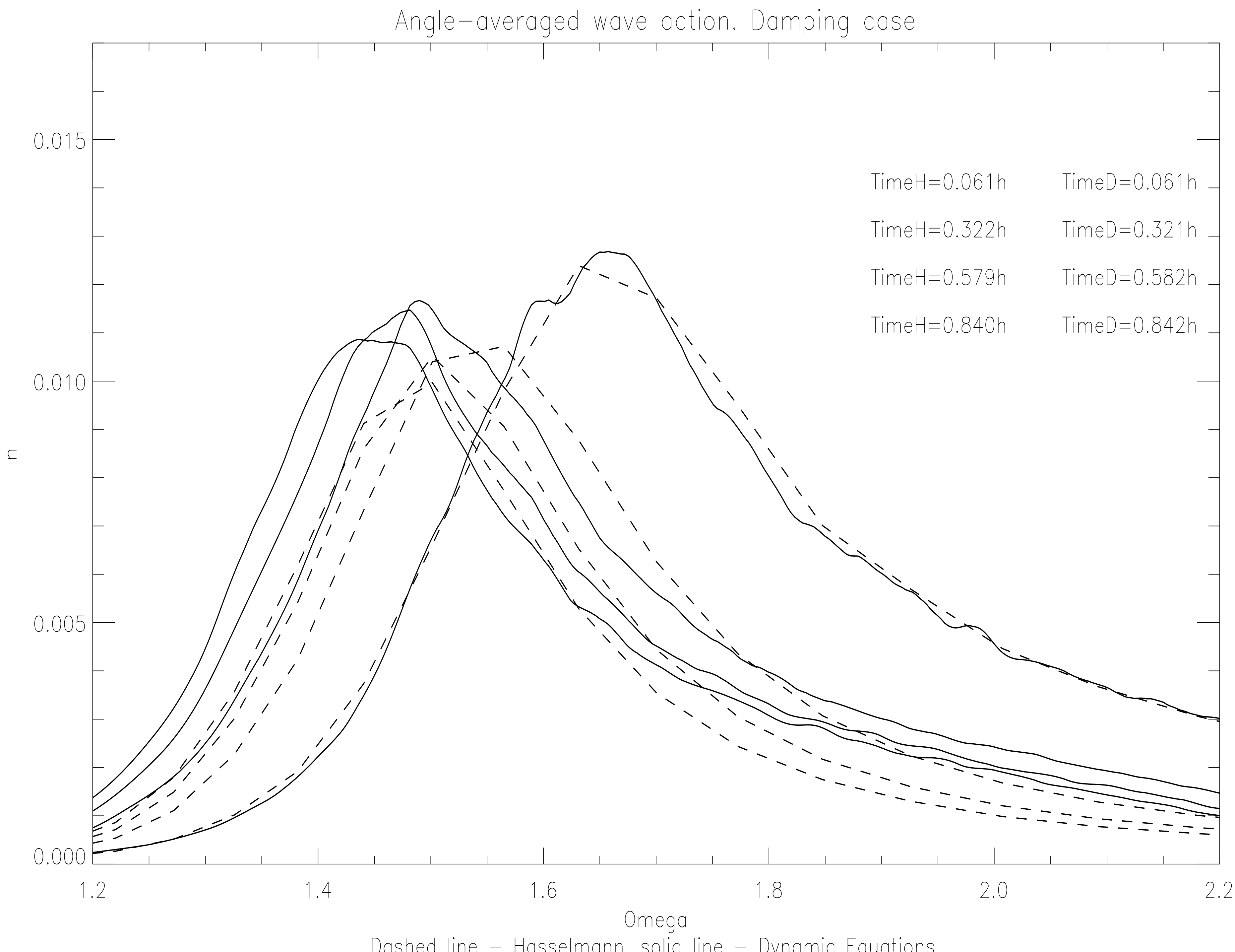
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10
ω
$|\eta_\omega|^2$
$|\eta_\omega|^2$
ω^{-4}



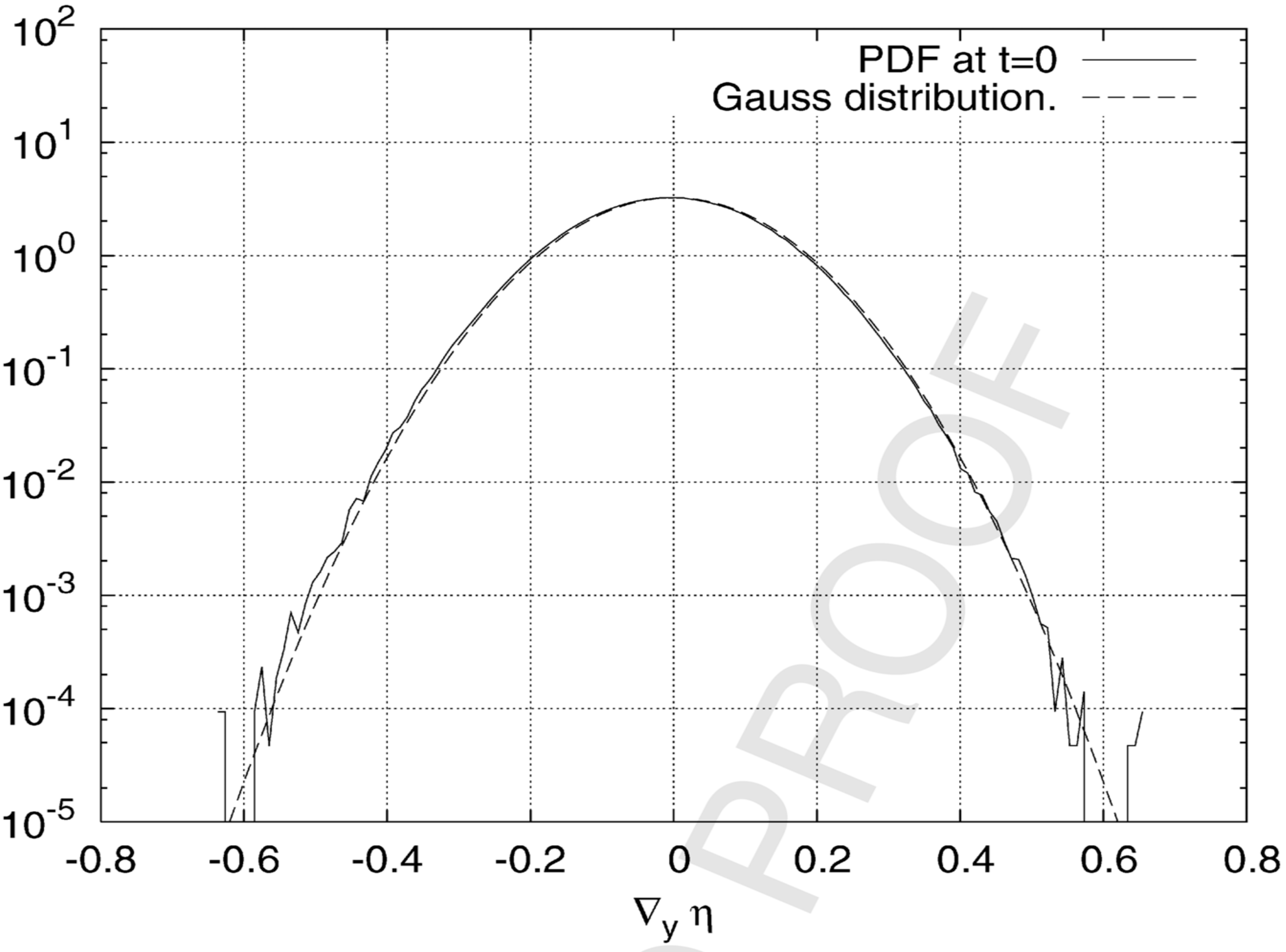
Angle-averaged wave action. Damping case
TimeH=0.061h
TimeD=0.061h
TimeH=0.324h
TimeD=0.321h
TimeH=0.579h
TimeD=0.582h
TimeH=0.840h
TimeD=0.842h
0.015
0.010
0.005
0.000
n
1.2
1.4
1.6
1.8
2.0
2.2
Omega
Dashed line - Hasselmann, solid line - Dynamic Equations

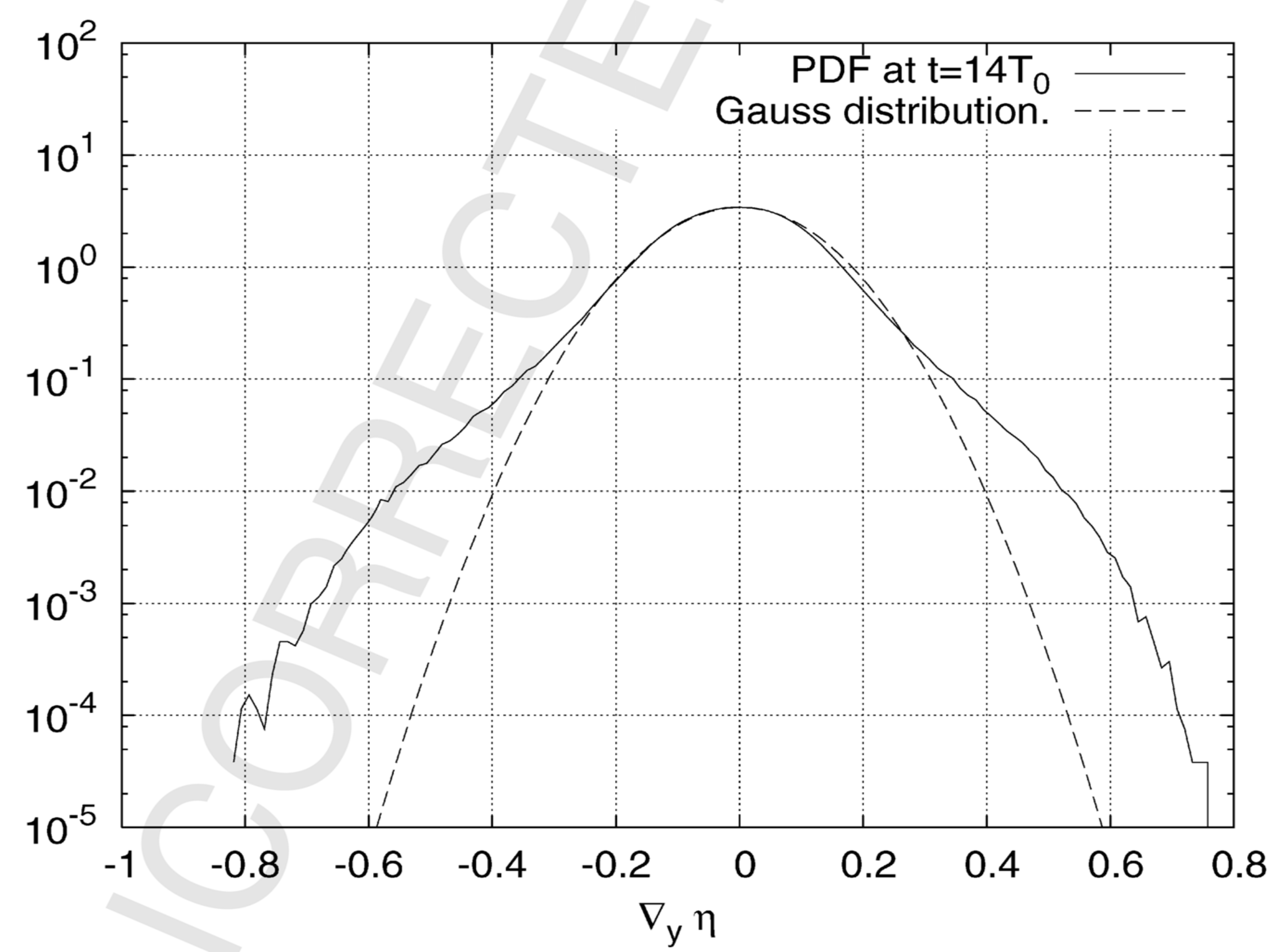


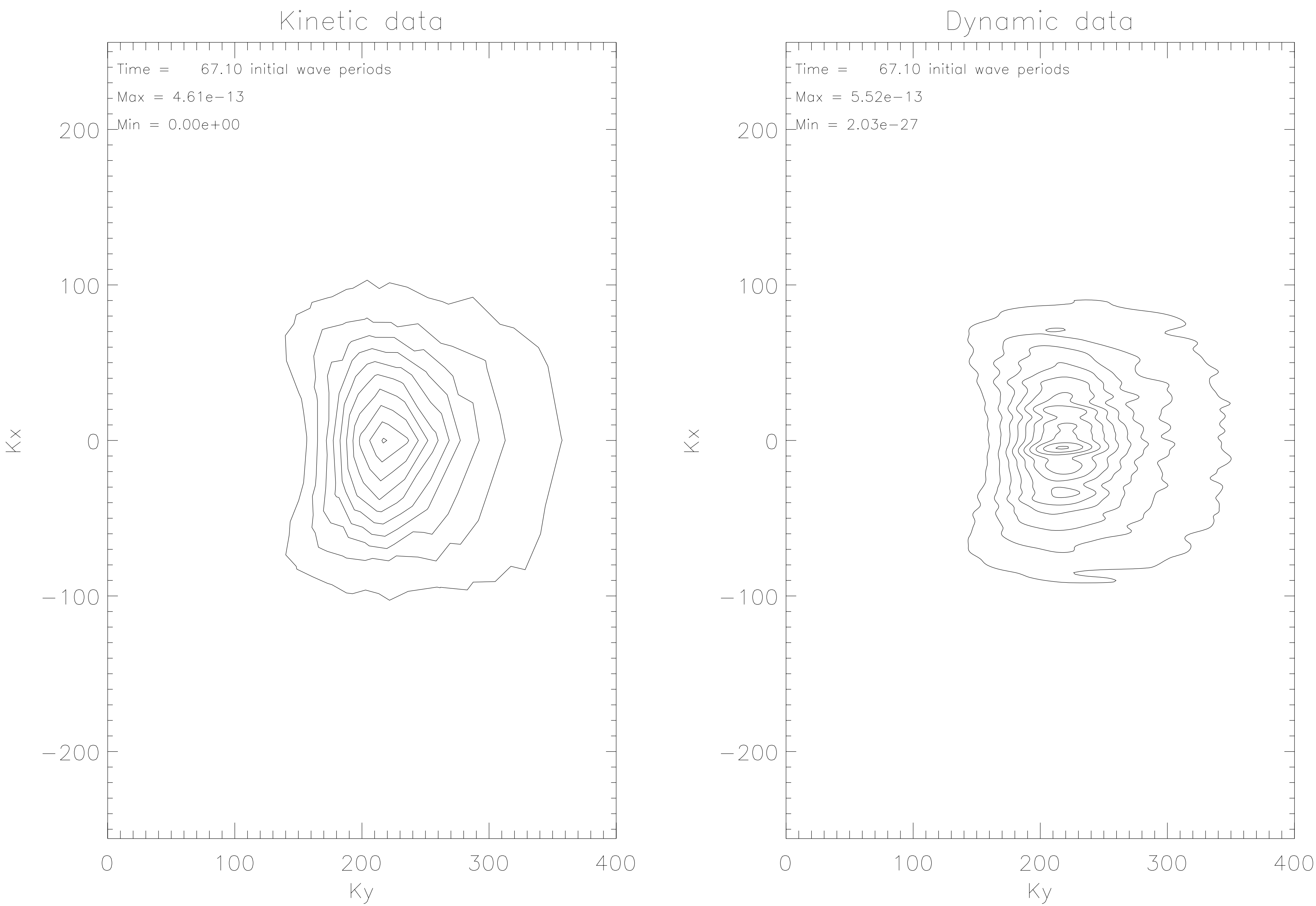
Angle−averaged wave action. Damping case
TimeH=0.061h
TimeD=0.061h
TimeH=0.324h
TimeD=0.321h
TimeH=0.580h
TimeD=0.582h
TimeH=0.841h
TimeD=0.842h
0.015
0.010
0.005
0.000
n
1.2
1.4
1.6
1.8
2.0
2.2
Omega
Dashed line − Hasselmann, solid line − Dynamic Equations

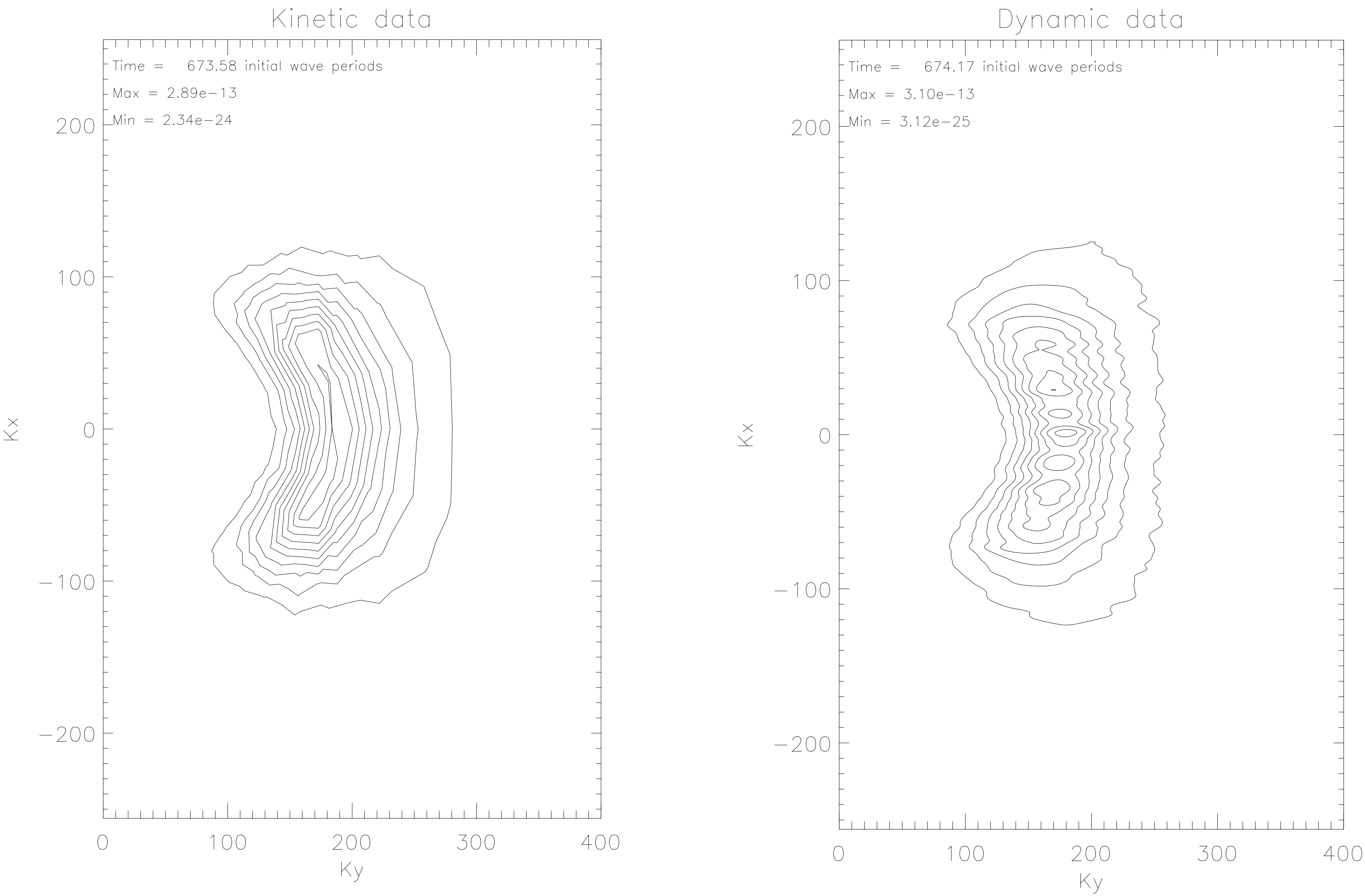


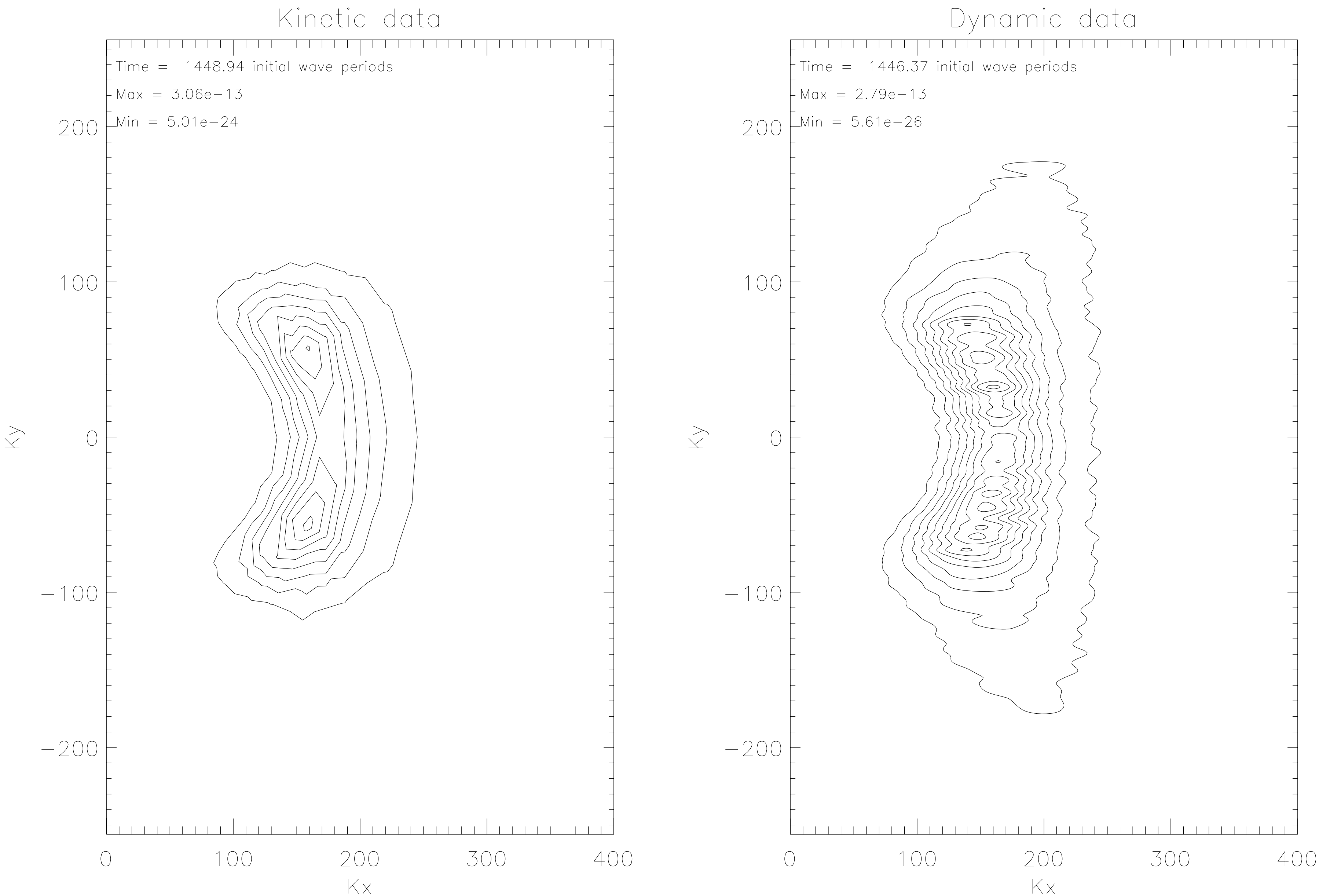
Angle−averaged wave action. Damping case
TimeH=0.061h
TimeD=0.061h
TimeH=0.322h
TimeD=0.321h
TimeH=0.579h
TimeD=0.582h
TimeH=0.840h
TimeD=0.842h
0.015
0.010
0.005
0.000
n
1.2
1.4
1.6
1.8
2.0
2.2
Omega
Dashed line − Hasselmann, solid line − Dynamic Equations


PDF at t=0
Gauss distribution.
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
∇y η

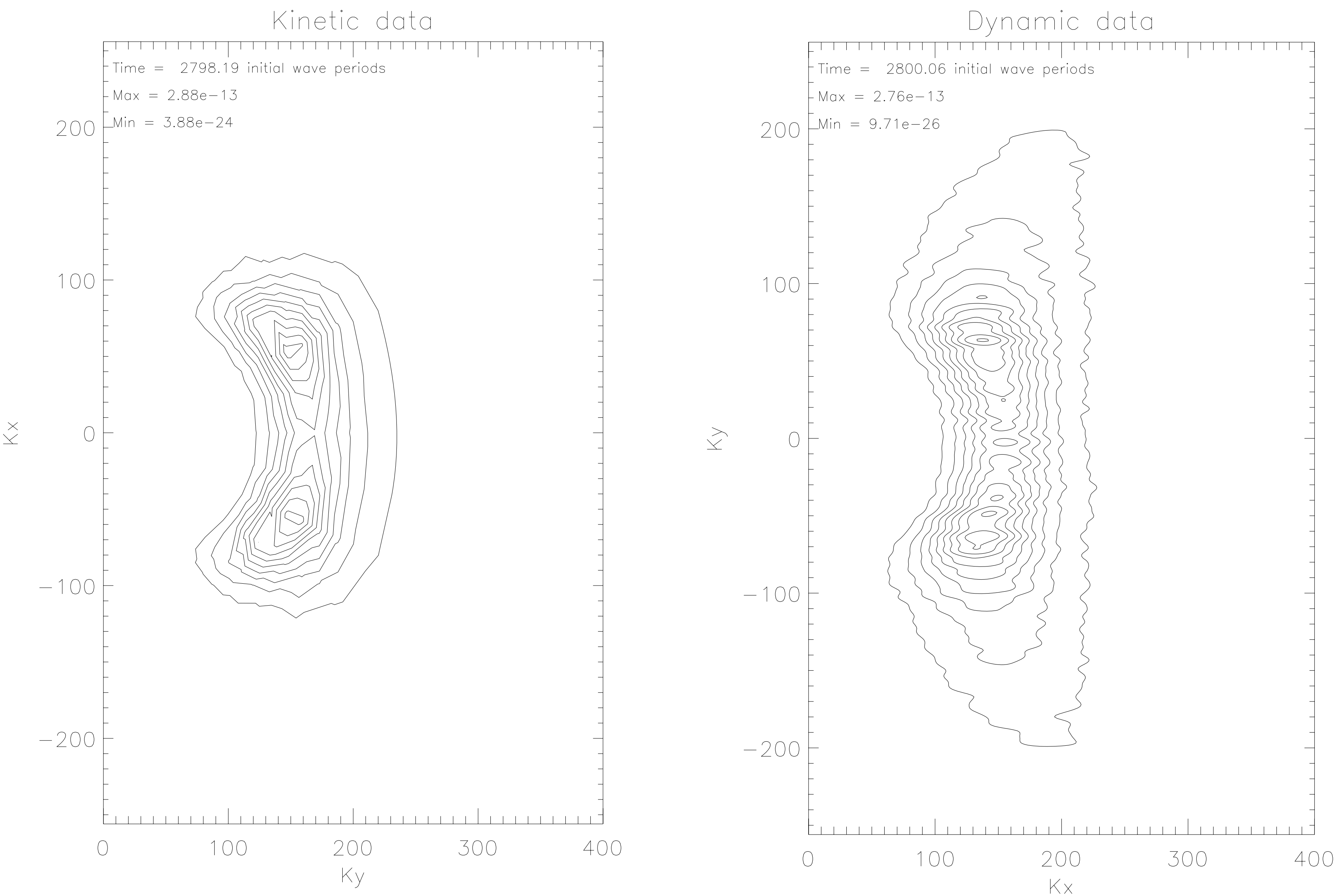

PDF at t=14T0
Gauss distribution.
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
∇y η


Kinetic data
Time = 67.10 initial wave periods
Max = 4.61e−13
Min = 0.00e+00
200
100
0
−100
−200
Kx
0
100
200
300
400
Ky
Dynamic data
Time = 67.10 initial wave periods
Max = 5.52e−13
Min = 2.03e−27
200
100
0
−100
−200
Kx
0
100
200
300
400
Ky

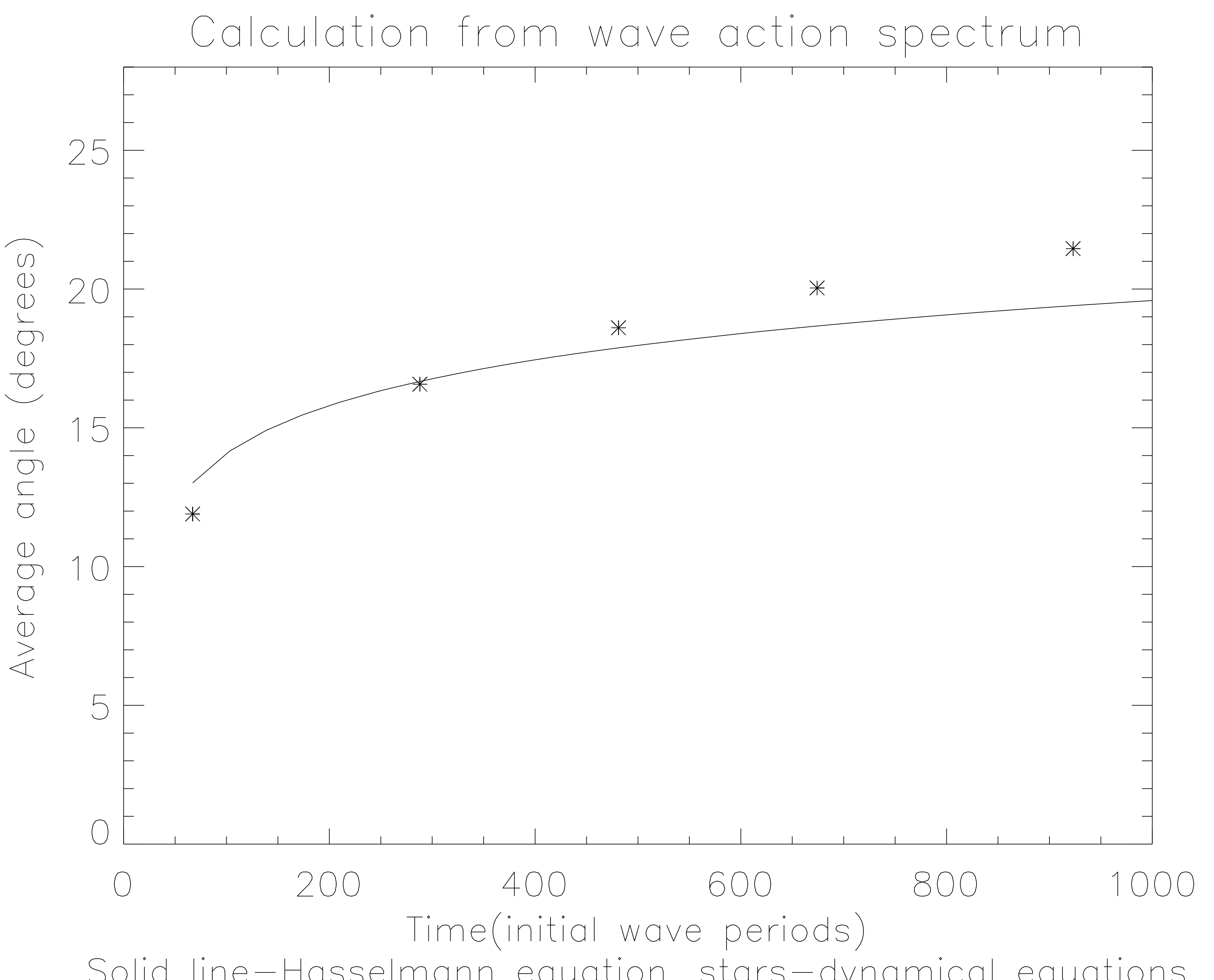

Kinetic data
Time = 673.58 initial wave periods
Max = 2.89e−13
Min = 2.34e−24
Kx
Ky
200
100
0
−100
−200
0
100
200
300
400
Dynamic data
Time = 674.17 initial wave periods
Max = 3.10e−13
Min = 3.12e−25
Kx
Ky
200
100
0
−100
−200
0
100
200
300
400



Kinetic data
Time = 1448.94 initial wave periods
Max = 3.06e−13
Min = 5.01e−24
Ky
Kx
200
100
0
−100
−200
0
100
200
300
400
Dynamic data
Time = 1446.37 initial wave periods
Max = 2.79e−13
Min = 5.61e−26
Ky
Kx



Kinetic data
Time = 2798.19 initial wave periods
Max = 2.88e−13
Min = 3.88e−24
Kx
Ky
200
100
0
−100
−200
0
100
200
300
400
Dynamic data
Time = 2800.06 initial wave periods
Max = 2.76e−13
Min = 9.71e−26
Ky
Kx

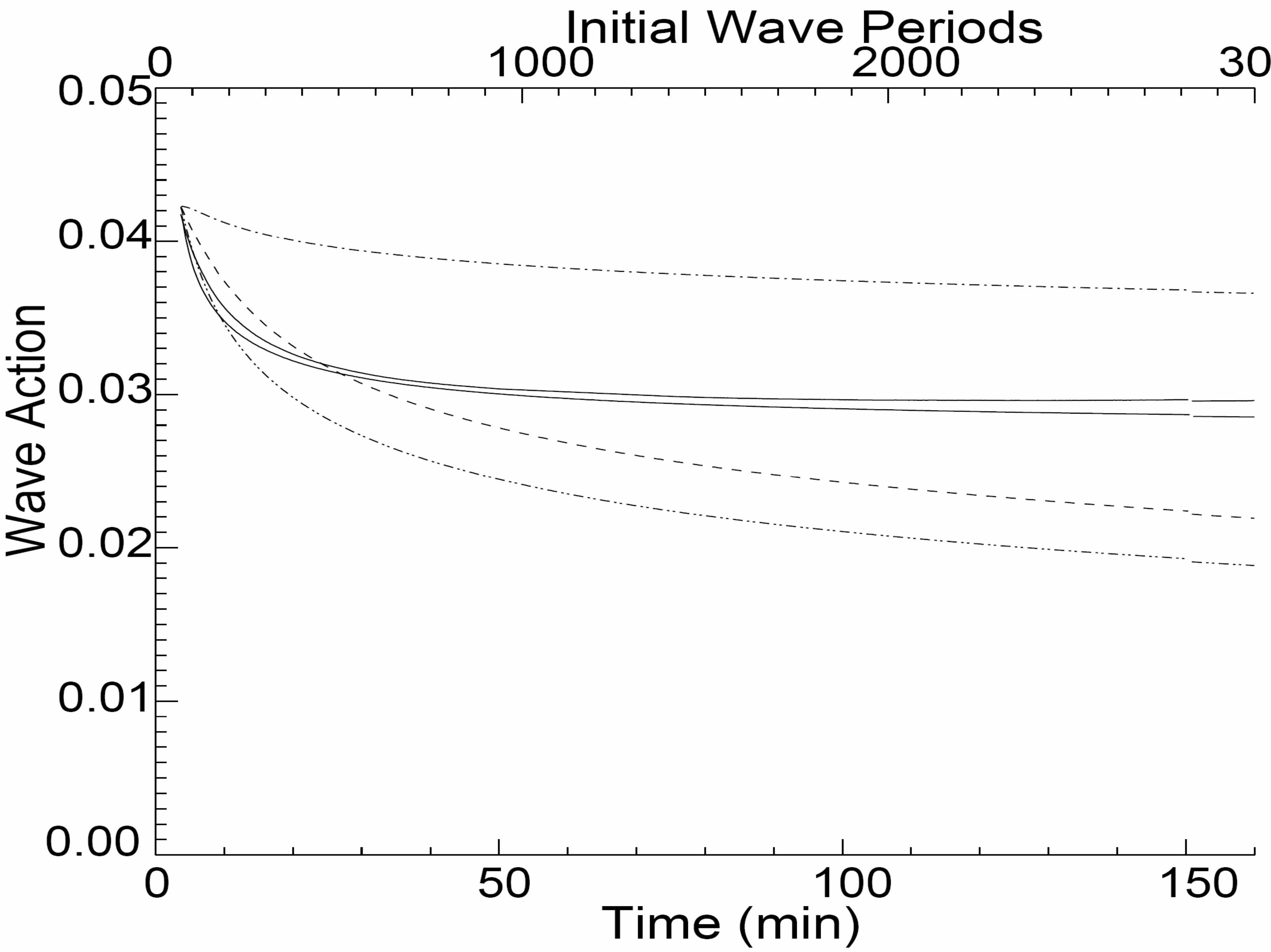

Calculation from wave action spectrum
Average angle (degrees)
0
5
10
15
20
25
0
200
400
600
800
1000
Time(initial wave periods)
Solid line−Hasselmann equation, stars−dynamical equations

New Dissipation Function:

$$S_{ds}(\omega,\theta) = -C_{ds}\left(1-\sigma+\sigma\frac{k}{\tilde{k}}\right)\left(\frac{\tilde{S}}{\tilde{S}_{PM}}\right)^{P}\tilde{\omega}\frac{k}{\tilde{k}}E(\omega,\theta)$$

$$\tilde{S} = \tilde{k}\sqrt{E_{tot}} \qquad \tilde{S}_{PM} = (3.02\cdot 10^{-3})^{1/2}$$

$$C_{ds} = 1.00\times 10^{-6}, \quad \delta = 0, P = 12$$



Initial Wave Periods
0
1000
2000
30
0.05
0.04
0.03
0.02
0.01
0.00
Wave Action
0
50
100
150
Time (min)

Conclusion:

- *Weak turbulence is confirmed through direct comparison of Hasselmann equation supplied with* ***New Dissipation Term*** *with dynamical equations.*

2. Experimental pools have to be longer that 200m to get the physics equivalent to the open ocean conditions.

Thanks for continuing support

to

US Army Corps of Engineers

and

Office of Naval Research