OPERATIONALISATION OF THE TSA FOR THE COMPUTATION OF NON-LINEAR FOUR-WAVE INTERACTIONS IN DISCRETE SPECTRAL WAVE MODELS

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1 INTRODUCTION

This paper describes the first steps towards the operationalisation of an approximate method for the computation of non-linear four-wave interactions in discrete wave spectra. This method, the Two Scale Approximation (TSA), was already described in Resio et al. (1992), but it took years to mature (cf. Resio and Perrie, 2007). It is a hybrid method using a combination of pre-computed exact non-linear transfer rates and correction terms. Computations by Resio and Perrie (2007) show that the TSA is able to approximate the exact non-linear transfer rate rather well for a wide range of spectra. The TSA is valid for both deep and shallow water. These promising results suggest that the TSA qualifies for inclusion in operational discrete spectral wave prediction models. In this way an acceptable balance might be achieved between computational requirements and accuracy. A successful implementation of the TSA in operational wave models might replace the Discrete Interaction Approximation (DIA), which was developed by Hasselmann et al. (1985). As described in Van Vledder et al. (2000), the DIA suffers from a number shortcomings that might be remedied by the TSA.

The basic feature of the TSA is to split an arbitrary input spectrum in two parts. The first part is referred to as the broadband structure which captures the main shape of the input wave spectrum and which can be described parametrically. The second part is the residual part, defined as the difference between the arbitrary wave spectrum and the parametric spectrum.

The non-linear transfer rate of the total wave spectrum is computed in two steps. In the first step the non-linear transfer rate for the broadband spectrum is obtained from a database with pre-computed exact transfer rates, for instance with the WRT method (Van Vledder, 2006). This part of the solution is referred to as the first scale. Using a database implies that only a limited number of spectral shapes are included in the database. In the second step the total transfer rate is determined by adding the contribution of so-called cross terms consisting of the residual spectral densities, pre-computed coupling coefficients and Jacobians. These cross terms can be considered as correction terms to the non-linear transfer rate of the broadband structure. This part of the solution is referred to as the second scale. Since not all cross terms are accounted for, some approximations are made in the TSA method.

The inclusion of the TSA in a discrete spectral wave model is of practical interest. To that end, the TSA should be written in subroutine form. The input of such a subroutine is the discrete wave spectrum in combination with the discrete frequencies (or wave numbers), discrete angles and water depth. The output of the subroutine consists of the non-linear transfer rate. Optionally, additional input and output arguments can be specified in such a way that they control the inner workings of the TSA and report on possible problems in the evaluation of the TSA.

The operationalisation of the TSA comprises transforming the research version of the TSA code into a generally applicable subroutine that can be implemented in any third-generation spectral wave model, like STWAVE, SWAN, WAM and WAVEWATCH. It is stressed that this paper is not about the quality of the TSA as an alternative to replace the DIA in operational wave models. Such aspects are addressed elsewhere and in subsequent studies about the performance of an operational TSA in discrete spectral wave models. In this paper the focus is on the operational aspects of the TSA.

The structure of this paper is as follows. Section 2 describes the TSA method and its resemblance with the WRT method. Its algorithmic implementation as a generally applicable subroutine is described in Section 3. Finally, Section 4 describes the further development of the TSA and possible optimizations.

2 DESCRIPTION OF THE TSA METHOD

The TSA method is based on the WRT-method for the computation of the non-linear transfer rate in a discrete wave spectrum (Tracy and Resio, 1982, Resio and Perrie, 1991). The WRT is based on Webb's (1978) choice of transformations to remove the δ-functions in the Boltzmann integral derived by Hasselmann (1962). Following Van Vledder (2006) the non-linear transfer rate or rate of change of the action density n_1 at wave number k_1 can be written as

$$\frac{\partial n_1}{\partial t} = \iint k_3 dk_3 d\theta_3 T\left(\boldsymbol{k}_1, \boldsymbol{k}_3\right) \tag{1}$$

in which the transfer function T is given by

$$\begin{aligned} T\left(\boldsymbol{k}_1, \boldsymbol{k}_3\right) = \iint d\boldsymbol{k}_2 d\boldsymbol{k}_4 \times G \times \delta\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4\right) \\ \times \delta\left(\omega_1 + \omega_2 - \omega_3 - \omega_4\right) \times \left[n_1 n_3 \left(n_4 - n_2\right) + n_2 n_4 \left(n_3 - n_1\right) \right] \end{aligned} \tag{2}$$

The term G is the coupling coefficient, which is a function of all four wave numbers involved in an interaction. The δ-functions reflect the resonance conditions which also ensure conservation of energy, action and momentum. As described in Van Vledder (2006) removing the delta-functions in (1) leads to the following expression for the function T

$$T\left(\boldsymbol{k}_1,\boldsymbol{k}_3\right)=\oint_s ds\times G\times J\times N_{1,2,3,4}\,. \tag{3}$$

The function T consists of a line integral over a closed locus in wave number space and where G, J and $N_{1,2,3,4}$ are functions of the locus coordinate s. The locus can be considered as the solution in wave number space of the resonance conditions for a given combination of the wave numbers $\boldsymbol{k}_1$ and $\boldsymbol{k}_3$. In (3) J is the Jacobian term given by

$$J=\left|\boldsymbol{c}_{g,2}-\boldsymbol{c}_{g,4}\right|^{-1} \tag{4}$$

The term $N_{1,2,3,4}$ is the product term of action densities

$$N_{1,2,3,4}=n_1n_3\left(n_4-n_2\right)+n_2n_4\left(n_3-n_1\right) \tag{5}$$

Details about the determination of the locus and the evaluation of (3) can be found in Van Vledder (2006).

As noted before, the basic feature of the TSA method is to split the spectrum into a broadband part b and a perturbation term p for all four wave numbers involved in an interaction according to

$$n_i=b_i+p_i \quad \text{for} \quad i=1,4 \tag{6}$$

Substitution of (6) into the product term of action densities $N_{1,2,3,4}$ leads to

$$\begin{aligned} N_{1,2,3,4} \;=\; & b_1b_3\left(b_4-b_2\right) + b_2b_4\left(b_3-b_1\right) + \\ & p_1p_3\left(p_4-p_2\right) + p_2p_4\left(p_3-p_1\right) + \\ & b_1b_3\left(p_4-p_2\right) + b_2b_4\left(p_3-p_1\right) + \\ & p_1p_3\left(b_4-b_2\right) + p_2p_4\left(b_3-b_1\right) + \\ & b_1p_3\left(b_4-b_2\right) + b_2p_4\left(b_3-b_1\right) + \\ & p_1b_3\left(b_4-b_2\right) + p_2b_4\left(b_3-b_1\right) + \\ & b_1p_3\left(p_4-p_2\right) + b_2p_4\left(p_3-p_1\right) + \\ & p_1b_3\left(p_4-p_2\right) + p_2b_4\left(p_3-p_1\right) \end{aligned} \tag{7}$$

The first line of Eq. (7) consists of action densities of the broadband spectrum. The second line contains action densities of the perturbation spectrum, whereas the other terms contain a varying mix of action densities from the broadband and perturbation spectrum. The complete transfer integral can be separated into the transfer rate of the broadband spectrum and seven additional contributions

$$\frac{\partial n_1}{\partial t}=B\left(\boldsymbol{k}_1\right)+\sum\nolimits_{j=2}^{8}\iint\oint_s N_j G\,J\,ds\,k_3 d\theta_3 dk_3 \tag{8}$$

In (8) the subscript j refers to the j^{th} row in Eq. (8) and $B(\boldsymbol{k}_1)$ is the exact transfer rate for the broadband spectrum. In Resio and Perrie (2007) it is argued that the terms in (7) containing the perturbation terms p_2 and p_4 can be omitted since they do not contribute significantly to the total transfer rate. A practical benefit of this assumption is that these terms are not easily known for a given decomposition of an arbitrary spectrum, whereas the other terms follow directly from the decomposition. Retaining the other terms yields

$$\begin{aligned} \tilde{N}_{1,2,3,4} \;=\; & b_1 b_3 (b_4 - b_2) \;+\; b_2 b_4 (b_3 - b_1) + \\ & b_2 b_4 (p_3 - p_1) \;+\; p_1 p_3 (b_4 - b_2) + \\ & b_1 p_3 (b_4 - b_2) \;+\; p_1 b_3 (b_4 - b_2) \end{aligned} \tag{9}$$

Substitution of (9) in Eq. (8) gives

$$\begin{aligned} \frac{\partial n_1}{\partial t} \;=\; & B(\boldsymbol{k}_1) \;+\; \iint \oint_s \left[b_2 b_4 (p_3 - p_1) + p_1 p_3 (b_4 - b_2) + b_1 p_3 (b_4 - b_2) + p_1 b_3 \right] JG ds k_3 dk_3 d\theta_3 \\ \;=\; & B(\boldsymbol{k}_1) \;+\; \iint \left[(p_3 - p_1) \oint_s b_2 b_4 JG ds \right] k_3 dk_3 d\theta_3 \\ & +\; \iint \left[(p_1 p_3 + b_1 p_3 + p_1 b_3) \oint_s (b_4 - b_2) JG ds \right] k_3 dk_3 d\theta_3 \end{aligned} \tag{10}$$

The terms $\oint_s b_2 b_4 JG ds$ and $\oint_s (b_4 - b_2) JG ds$ depend on the wave number combination $(\boldsymbol{k}_1, \boldsymbol{k}_3)$ but not on the actual action density terms. Therefore, they can be pre-computed. This is done using an adapted version of the WRT method. Following Resio and Perrie (2007) these terms can be written as a pumping term (product of the densities at the wave numbers $\boldsymbol{k}_1$ and $\boldsymbol{k}_3$) and a diffusive term (involving differences in densities of the wave numbers $\boldsymbol{k}_1$ and $\boldsymbol{k}_3$):

$$\begin{aligned} \Lambda_p (\boldsymbol{k}_1, \boldsymbol{k}_3) &= \oint_s (b_4 - b_2) JG ds \big|_{\boldsymbol{k}_1, \boldsymbol{k}_3} \\ \Lambda_d (\boldsymbol{k}_1, \boldsymbol{k}_3) &= \oint_s (b_2 b_4) JG ds \big|_{\boldsymbol{k}_1, \boldsymbol{k}_3} \end{aligned} \tag{11}$$

Combining the various equations leads to the following formulation of the TSA

$$\begin{aligned} \frac{\partial n_1}{\partial t} \;=\; & B(\boldsymbol{k}_1) \;+\; \iint (p_3 - p_1) \Lambda_d (\boldsymbol{k}_1, \boldsymbol{k}_3) k_3 dk_3 d\theta_3 \\ & +\; \iint (p_1 p_3 + p_1 b_3 + b_1 p_3) \Lambda_p (\boldsymbol{k}_1, \boldsymbol{k}_3) k_3 dk_3 d\theta_3 \end{aligned} \tag{12}$$

Equation (12) reveals the formal structure of the TSA. Similar to the WRT method, this formulation is valid in deep and shallow water. For any given wave number $\boldsymbol{k}_1$ a loop over all spectral components $\boldsymbol{k}_3$ is to be made. The contributions to the transfer rate consist of products of action densities of the broadband spectrum and the perturbation spectrum, multiplied with tabulated data stored in the matrices Λ_d and Λ_p. The main difference with the WRT method is the absence of the repeated integration along the loci for each $\boldsymbol{k}_1$-$\boldsymbol{k}_3$ wave number combination, which saves a considerable amount time.

3 OPERATIONALISATION OF THE TSA

Discretisation

An important step in the operationalisation of the TSA is to discretise expression (12) such that it can be applied in a discrete spectral wave model. It is assumed that the wave spectrum is given in terms of a discrete energy density function of the discrete frequencies f_i (for i=1,N_f) and directions θ_j (j=1,N_θ) with a constant spacing $\Delta\theta$. An additional requirement is that the frequencies (or wave numbers) are distributed geometrically, viz. subsequent frequencies are related according to $f_{i+1}=(1+\delta) f_\mathrm{i}$, with δ=0.1 typically. In practice the proper Jacobian transformation between wave number space based action density spectra and frequency-direction energy density spectra must be applied since existing operational wave model use different conventions for wave spectra.

Based on expression (12) the non-linear transfer rate at a certain discrete wave number $\left(k_{i_{k1}}, \theta_{j_{j1}}\right)$ is expressed as:

$$\begin{aligned}\Delta n\left(k_{i_{k1}}, \theta_{j_{k1}}\right) = {} & B\left(k_{i_{k1}}, \theta_{j_{k1}}\right) \\ & + \sum_{i_{k3}=1}^{N_k} \sum_{j_{k3}}^{N_\theta} \left(p_{3 i_{k3}, j_{k3}} - p_{1 i_{k3}, j_{k3}} \right) \Lambda_a \left(i_{k1}, j_{k1}, i_{k3}, j_{k3}\right) k_{i_{k3}} \Delta k_{i_{k3}} \Delta\theta \\ & + \sum_{i_{k3}=1}^{N_k} \sum_{j_{k3}}^{N_\theta} \left(p_{1 i_{k1}, j_{k1}} p_{3 i_{k3}, j_{k3}} + b_{1 i_{k1}, j_{k1}} p_{3 i_{k3}, j_{k3}} + p_{1 i_{k1}, j_{k1}} b_{3 i_{k3}, j_{k3}} \right) \Lambda_b \left(i_{k1}, j_{k1}, i_{k3}, j_{k3}\right) k_{i_{k3}} \Delta k_{i_{k3}} \Delta\theta\end{aligned} \tag{13}$$

The above expression is the basic computational scheme of the TSA. Thus, for every spectral bin a loop is made over all spectral bins to compute the contribution of the cross terms using pre-computed data from the modified WRT method. In expression (13) the terms B, Λ_d and Λ_p are two- and four-dimensional matrices containing pre-computed results for a given broadband action density spectrum $b(\boldsymbol{k})$. These matrices are linked to each other since they are all based on the same broadband spectrum.

It is noted that expression (13) can be optimized considerably. Many terms can be neglected since their contributions to the total transfer rate are insignificant. Expression (13) contains inner loops over all possible spectral bins ($k_{3\mathrm{i}}$,$\theta_{3\mathrm{j}}$). Similar to the WRT method, many contributions can be filtered out when the 'distance' in wave number space

between the wave numbers $\boldsymbol{k}_1$ and $\boldsymbol{k}_3$ is larger than a certain threshold value. Such a limitation of integration ranges can be formalized by introducing the normalized variables k_* and θ_* according to (cf. Resio and Perrie, 2007)

$$\begin{aligned} k_* &= \frac{k_1 - k_3}{k_p} \\ \theta_* &= \left[\theta_1 - \theta_2\right] \end{aligned} \tag{14}$$

The brackets in the directional term indicate that periodicity of directions is accounted for. A disadvantage of (14) is its dependency on the peak wave number k_p. This led Van Vledder (2006) to apply a local wave number criterion based on the ratio of the wave numbers k_1 and k_3, thus $k_*=k_1/k_3$. Both methods can be used to reduce the computational requirements of the TSA while retaining sufficient accuracy. In the present phase of the operationalisation of the TSA these definitions of the variables k_* and θ_* and their integration ranges are not yet determined.

As with any approximate method the accuracy of the TSA degrades as the arbitrary spectrum deviates more and more from the broadband spectrum. A straightforward solution to this problem is to have a set of many broadband spectra with associated pre-computed non-linear transfer rates and matrices Λ_d and Λ_p. These pre-defined spectra (and related transfer rate and matrices) must cover a wide range of spectral shapes regarding e.g. peak frequency, scale factor, peakedness and directional spreading to ensure accuracy of the TSA. It is therefore of interest to determine the number of required spectral shapes to enable the TSA to produce accurate answers.

Fortunately, scaling rules exist that limit the number of required spectral shapes. It can be shown that for spectra of the form

$$E\left(f,\theta\right) = \alpha f_p^{-n} \Psi\left(\upsilon,\theta\right) \tag{15}$$

with $\nu=f/f_p$, the nonlinear transfer rate of energy density spectra can be expressed as

$$S_{nl}\left(f,\theta\right) = \alpha^3 f_p^{11-3n} \Omega\left(\upsilon,\theta\right) \tag{16}$$

In addition, rotational symmetry exists for two similar spectra that only differ in their mean directions. In the case of spectra with two different mean directions $\theta_{m,1}$ and $\theta_{m,2}$, the non-linear transfer rate is rotated over the angle $\Delta\theta=\theta_{m,1}-\theta_{m,2}$. Applying these rules to two similarly shaped spectra, but with different parameters α, f_p and θ_m gives the following relationship valid for JONSWAP type spectra with either an f^{-4} or an f^{-5} spectral tail[1]:

[1] JONSWAP spectra can be written in the general form $\alpha g^2(2\pi)^{1-m} f^{-m} f_p^{m-5} \exp[-(m/n)(f/f_p)^{-n}] \Gamma$ with Γ the peak enhancement factor and m the power of the spectral tail. The f_p^{-m}-term is required to keep the proper dimensions and to ensure that the peak of the spectrum occurs at $f=f_p$. A useful discussion on this subject can be found in Holthuijsen (2007).

$$S_{nl}^{(2)}(f,\theta) = S_{nl}^{(1)}\left(f\frac{f_{p1}}{f_{p2}},\theta-\Delta\theta\right)\left(\frac{\alpha_2}{\alpha_1}\right)^3\left(\frac{f_{p2}}{f_{p1}}\right)^{-4} \tag{17}$$

The transformation rules and rotational symmetry reduce the number of required pre-computed spectra and related transfer rates and matrices. The way in which these rules are applied in a subroutine version of the TSA is explained in the next Section.

An example of the scaling of non-linear transfer rates for two similar spectra but with different scale factors and peak frequencies is shown in Figure 1. For higher frequencies the scaling is not perfect. This is due to the finite frequency range for which the transfer rate is computed.

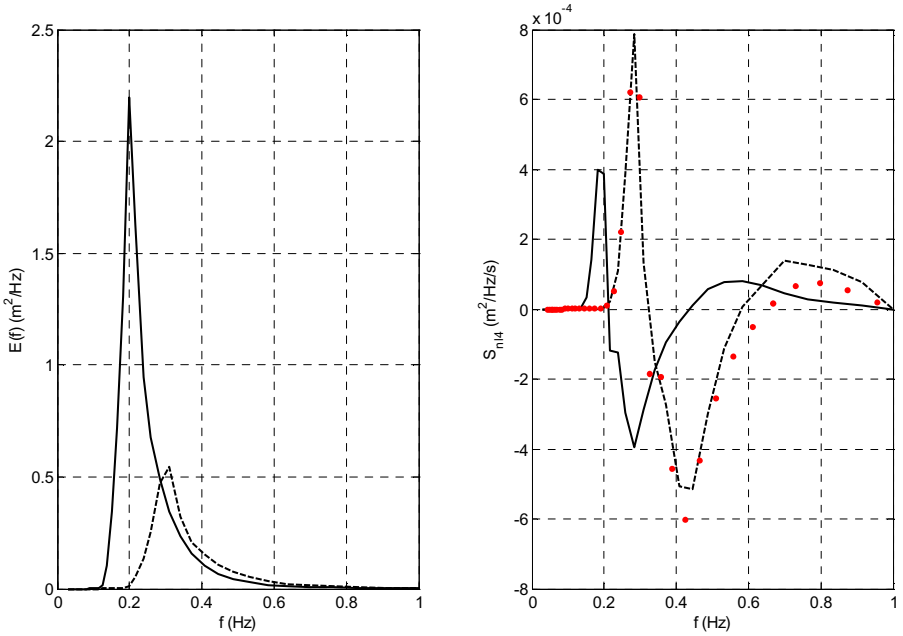

Figure 1: JONSWAP spectra (γ=2) with peak frequencies of 0.2 Hz and 0.3 Hz and related non-linear transfer rate. The scaled transfer rate is indicated by the red dots.

In shallow water, the scaling with the peak frequency does not apply since the shape of the transfer rate also depends on the water depth. It is therefore necessary to have a set of pre-computed spectra that are characterised by different values of $k_m h$ in which k_m is the mean and h is the water depth. The peak wave number k_p is considered to be less suited since it estimation is not robust enough, especially in the case of multi-peaked spectra.

Structure of the operational TSA

The transition of the TSA from a research code into a generally applicable code requires a rethinking of its structure. A characteristic of the research code used by Resio and Perrie (2007) is the emphasis on the TSA as a means to compute the non-linear transfer rate using many pre-set variables and a pre-set loop structure. The computation of additional source terms to simulate the evolution of the wave spectrum comes in second place.

For an operational version of the TSA the wave model comes in the first place, which needs separate subroutines to compute source terms for a given wave spectrum and environmental variables as wind, depth and current.

An important requirement of a generally applicable routine is that the number of frequencies and directions must be kept flexible. In addition, it must be possible to specify certain settings of the TSA regarding the extent of the inner loops, the computation of the spectral characteristics of the input spectrum, the decomposition of an arbitrary spectrum into a broadband spectrum and a perturbation spectrum, options for the search for the best-fitting broadband structure and location and format of the pre-computed broadband transfer rates and related matrices. Further, a convention for naming the pre-computed data files must be determined. Finally, the TSA is programmed in Fortran 90 to make use dynamical allocation memory.

The subroutine version of the TSA uses as basic input arrays with the discrete frequencies (or wave numbers), the directions, energy density spectrum (or action density spectrum) and the water depth. The output of the TSA routine consists of the nonlinear transfer rate at all spectral bins. In practise, a simple interface will be necessary to transform between the different conventions of wave model spectra in use by the various operational wave models like WAVEWATCH, STWAVE or SWAN and to reverse the indexing of frequency and direction bins if necessary. Additional input arguments comprise settings about the inner workings of the TSA. An additional output argument comprises of an error indicator.

The basic steps in the TSA subroutine are as follows:

- Determination of the characteristics of the input wave spectrum, viz. the peak frequency f_p, the energy contents E or scale factor α, the mean direction θ, the peakedness γ and the directional spreading σ. An example of fitting a parametric JONSWAP spectrum to an arbitrary input spectrum is shown in Figure 2. In this example the peak enhancement factor of the best-fitting JONSWAP spectrum is 1.15.
- For shallow water applications the above list needs to be extended with the dimensionless water depth $k_m h$, where k_m is a mean wave number and h the water depth. These parameters are needed to search the best fitting broadband spectrum and for the scaling of the nonlinear transfer rate. This list may need to be extended with other shape characteristics when further tests indicate relevant dependencies;
- Search the best corresponding broadband spectrum in the database represented by its peakedness γ, directional spreading σ and for shallow water the dimensionless water depth $k_m h$. As noted before, this list may need to be extended with other spectral characteristics. In Figure 2, a peakedness factor of 1.15 was found. In

general, this factor will differ from the set of pre-computed peak enhancement values of the broad-band spectra. In such a situation, the nearest value, e.g. 1, will be taken. The same applies for the other characteristics of the pre-computed broad-band spectra;

- Read the broadband exact non-linear transfer rate and related matrices from file. These data can be read in formatted or unformatted form. Formatted file *i/o* is useful for testing purposes. In future versions of the TSA unformatted file *i/o* will be implemented since this is much faster than formatted file *i/o*.
- Split the input spectrum into a broadband spectral form and the perturbation term. This procedure is illustrated in Figure 2. The characteristics of the broadband frequency spectrum can be the JONSWAP parameters f_p, α, γ. The directional spreading can be based on a $\cos^{2s}([\theta-\theta_m]/2)$ directional distribution, but other directional distribution can also be chosen. Further tests are needed to determine the optimal shape of the parameterized directional distribution;
- Apply scaling laws and directional transformation to obtain the proper transfer rate for the broadband spectrum and perturbation terms. An important consideration is that the peak frequency scaling of the spectra should be based on the discrete frequencies. Since the frequencies (and wave numbers) are geometrically spaced, a scaling of the frequency axis is similar to shift the energy (or action) densities over a discrete number of bins. Similar considerations apply to the angular rotation of the spectra. These rotations should be restricted to multiples of the directional step $\Delta\theta$. These additional requirements avoid the introduction of interpolation errors. It is noted that this technique is also used in the EXACT-NL model (Hasselmann and Hasselmann, 1985).
- Evaluate TSA correction terms and add these results to obtain the non-linear transfer rate for the transformed arbitrary input spectrum, as splitted in a broadband part and a perturbation part.
- The last step of the TSA subroutine is to transform the computed non-linear transfer rate back to the original frequencies and directions.

The TSA subroutine will also contain an error indicator as output variable to point to specific errors that might have occurred during the evaluation of the TSA. For instance, the program execution may stop when the pre-computed non-linear transfer rates matrices cannot be found because the path is invalid. Catching error messages enables the host program to take appropriate action. Such a procedure is required for any robust operational wave model.

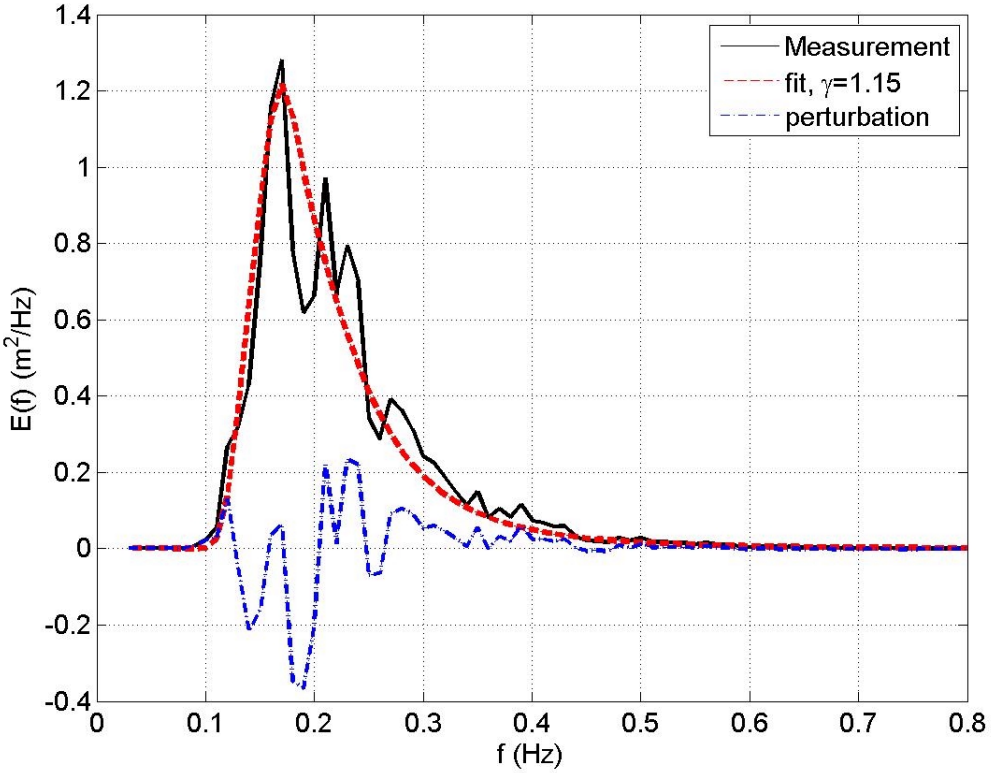

Figure 2: Measured frequency spectrum, fitted spectrum and perturbation spectrum.

4 FURTHER DEVELOPMENT OF THE TSA

Preparations for the TSA

An important feature of the TSA is the use of pre-computed non-linear transfer rates and related matrices for a range of parametric spectra. As noted before their peakedness and directional spreading will initially characterize these spectra. For shallow water the non-dimensional water depth $k_m h$ will be used. The extent and resolution of these characteristics is the subject of sensitivity studies in the further development of the operational TSA.

An essential requirement in the set-up of these pre-computed non-linear transfer rates and related matrices is that the discretisation of the frequencies and directions should be identical to those used by the host program. In addition, the power of the parametric spectral tail should also be equal to those of the host model. The computation of the pre-computed non-linear transfer rates should be performed with an adapted version of the WRT method. Such versions were developed by Resio and Perrie (2007) and by the author in the framework of this study using the WRT method described in Van Vledder (2006).

Testing the TSA subroutine
The implementation and operational use of the TSA is performed in steps, in which each steps adds further functionality. The first step in the development of the TSA was to make a modified version of the WRT method to generate the non-linear transfer rate and related matrices for a given parametric spectrum. In addition, a subroutine was set-up for reading an energy density spectrum with the same characteristics as the parametric spectrum that was used to generate the database with the non-linear transfer rate and related matrices. In this way the consistency between all elements of the TSA was tested for the first scale. In the next step a small perturbation to the input spectrum was made to test the effectiveness of the correction terms, which is the essence of the second scale of the TSA.

The second phase of testing the TSA is related to test the scaling of similar spectra. In this phase the following aspects are tested:

- α-scaling for similar spectra but with different energy contents;
- frequency scaling for similar spectra but with different peak frequencies;
- rotational transformation for similar spectra but with different mean directions.

The third phase is to test the effectiveness of the TSA when the arbitrary input spectra deviates in shape from the parametric spectra for which non-linear transfer rates and related matrices have been computed. Resio and Perrie (2007) carried out such tests, but it is essential to repeat these tests and confirm their findings.

After completion of these tests the last and most important phase of the testing will start, viz. its applicability in operational wave models and its added value compared to the DIA. Academic tests on individual (parametric) spectra are no guarantee that the TSA will also work for spectra that occur during a wave model run. Therefore, the following aspects will be addressed in this phase:

- The reproduction of realistic growth curves and their similarity with a wave model version using an exact computation of the non-linear transfer rate. These tests are needed to show that the system of equations (i.e. the wave action balance equation using the TSA as a subroutine) produces stable results.
- Its ability to produces (more peaked) spectral shapes than the DIA and the similarity of spectral shapes obtained with a model version using an exact computation of the non-linear transfer rate.
- The inclusion of the diagonal term for use in the implicit integration scheme of the WAM model.
- Overall model performance in academic and field cases. Here, model performance is related to integral wave parameters and spectral shapes. Interesting academic cases are the wave model response after a sudden wind shift or a slanting fetch situation. Interesting field cases should be taken from well-documented model studies, e.g. those in Lake George, Australia or at Duck, NC, USA.

Speeding up the TSA
The basic computational method of the TSA method described in this paper consists of a double loop over all wave numbers, scaling techniques and data transfer from pre-computed non-linear transfer rates and related matrices. In the preliminary phase of subroutine development efficiency generally has not the highest priority, since the proper workings of all elements of the TSA are crucial. In this phase of development the following potential optimisations were identified:

- Optimization of internal loops, will boils down to determine the optimal range of integration of the non-dimensional parameters k_* and θ_*. This will also affect the size of the pre-computed matrices;
- A useful property of the T-function is its symmetry: $T(\boldsymbol{k}_1, \boldsymbol{k}_3) = -T(\boldsymbol{k}_3, \boldsymbol{k}_1)$. This property allows computing only half of all possible combinations of wave numbers, and symmetric storing of the contributions to the non-linear transfer rate can be used.
- The present implementation use data transfer using formatted (ASCII) data files to enable easy inspection of the data. The speed of the TSA can be improved by replacing the data format by unformatted (binary) data files.

5 CONCLUSION

The TSA is a promising technique for the evaluation of the non-linear transfer rate in discrete spectral wave models. It is expected that the TSA provides an economic alternative to the exact computation of the non-linear transfer rate, such that it may replace the DIA as the working horse for the evaluation of these interactions in operational wave models. Various studies are under way to determine its applicability in operational wave models.

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