ON EXPERIMENTAL JUSTIFICATION OF WEAKLY TURBULENT NATURE OF GROWING WIND SEAS

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1. WEAKLY TURBULENT LAW OF WIND-WAVE GROWTH

The role of nonlinear transfer in dynamics of wind-driven waves is a subject of heated discussion for last years. The result of this discussion gives a key to burning problems of wave modelling: what constituents of wind wave balance should be attacked first both theoretically and experimentally? Recent studies (Resio *et al.*, 2004; Zakharov, 2005; Badulin *et al.*, 2005*b*) shows definitely the leading role of nonlinear transfer for growing wind seas. Effective asymptotic models of wind-driven seas can be proposed in this case. The asymptotic model of growing wind sea has been presented recently as the split balance model (Badulin *et al.*, 2005*a*, 2006, 2007). Within the model the spectral evolution is described by conservative kinetic equation (Hasselmann, 1962) for wave action spectral density $N(\mathbf{k}, t)$

$$\frac{\partial N_{\mathbf{k}}}{\partial t} + \nabla_{\mathbf{k}}\omega_{\mathbf{k}}\nabla_{\mathbf{r}}N_{\mathbf{k}} = S_{nl}\left[N(\mathbf{k})\right] \tag{1}$$

that does not contain generation and dissipation term. The specific 'boundary condition' — closure equation for total wave action and net wave input

$$\langle\frac{\partial N_{\mathbf{k}}}{\partial t} + \nabla_{\mathbf{k}}\omega_{\mathbf{k}}\nabla_{\mathbf{r}}N_{\mathbf{k}}\rangle = \langle S_{in} + S_{diss}\rangle \tag{2}$$

makes the model physically and mathematically correct. The feature of the model is the split of key constituents: nonlinear transfer term S_{nl} and terms of wave input S_{in} and dissipation S_{diss}. Such split allows for exploiting the riches of the theory of weak turbulence (Zakharov *et al.*, 1992): in particular cases of duration- and fetch-limited growths the model (1,2) has families of self-similar solutions which are direct analogues of the classic Kolmogorov-Zakharov solutions (Zakharov & Filonenko, 1966; Zakharov, 1966; Zakharov & Zaslavsky, 1982). While the classic Kolmogorov-Zakharov solutions are isotropic, not localized in frequency and have infinite total energy, the new self-similar solutions correspond to power law dependence of total energy ε and characteristic frequency ω_* on time t (or fetch x), i.e.

$$\varepsilon(t) = \varepsilon_0 t^{p_\tau}; \qquad \omega_*(t) = \omega_0 t^{-q_\tau}, \tag{3}$$

$$\varepsilon(x) = \varepsilon_0 x^{p_\chi}; \qquad \omega_*(x) = \omega_0 x^{-q_\chi}. \tag{4}$$

In (3,4) we intentionally put dimensional time, fetch and the corresponding coefficients ω_0, ε_0. Within the split balance model a traditional wind speed scaling seems to be, at least, strange as far as wind speed does not appear in the model (1, 2) explicitly.

Existence of a family of duration-limited solutions which are anisotropic and localized in frequency has been demonstrated by Badulin *et al.* (2005*b*). The extensive numerical study showed robustness of self-similarity features of wind-driven waves in a wide range of initial conditions for different parameterizations of wave input. Moreover, the robustness of wave growth has been demonstrated in terms of the basic relationship of weakly turbulent Kolmogorov's spectra — the relationship of spectral density and spectral flux. For total energy and net energy input of asymptotic self-similar solutions of the Hasselmann equation this relationship has been presented in the following form (Badulin *et al.*, 2007)

$$\frac{\varepsilon\,\omega_p^4}{g^2} = \alpha_{ss}\left(\frac{\omega_*^3\,\mathrm{d}\varepsilon/\mathrm{d}t}{g^2}\right)^{1/3} \tag{5}$$

Full derivative of total energy $d\varepsilon/dt$ (net wave input) reflects independence of the relationship on particular scenario of wind-wave development (duration- or fetch-limited). Self-similarity parameter α_{ss} depends, evidently, on definition of the frequency scale ω_*. Additionally, there is a dependence of α_{ss} on exponents $p_{\tau(\chi)}$, $q_{\tau(\chi)}$ in (3,4). Fortunately, this dependence is relatively weak (Badulin *et al.*, 2007). Thus, the relationship (5) can be treated as weakly turbulent growth law of wind-driven sea irrespectively to power-law dependencies (3,4).

Note, that so far the only 'observable' manifestation of weak turbulence of wind-driven seas was seen in high-frequency spectral tails (e.g. Resio *et al.*, 2004): the frequency spectra appear to be close to dependence ω^{-4} as predicted by Zakharov & Filonenko (1966). In this paper we propose the new evidence of the weakly turbulent nature of wind-driven waves basing on the growth law (5). We show that weak turbulence mechanisms control not only a notorious 'transparency range' of wave spectra but integral properties of wind-wave spectra: total energy ε and characteristic frequency ω_*.

The idea of the experimental verification of theoretical relationship (5) comes from 'experimental tradition' to parameterize wave growth by power-law fits similarly to (3,4). The feature of these experimental parameterizations is in the wind speed scaling. The resulting non-dimensional expressions look as follows (cf. 3,4)

$$\tilde{\varepsilon}(\tau) = \tilde{\varepsilon}_0 \tau^{p_\tau}; \qquad \tilde{\omega}_*(\tau) = \tilde{\omega}_0 \tau^{-q_\tau}, \tag{6}$$

$$\tilde{\varepsilon}(\chi) = \tilde{\varepsilon}_0 \chi^{p_\chi}; \qquad \tilde{\omega}_*(\chi) = \tilde{\omega}_0 \chi^{-q_\chi}. \tag{7}$$

where non-dimensional values are defined in a standard way

$$\tilde{\omega} = \omega U_h/g; \quad \tilde{\varepsilon} = \varepsilon g^2/U_h^4; \quad \chi = gx/U_h^2; \quad \tau = gt/U_h \tag{8}$$

U_h is wind speed at a reference height (or its substitute — friction velocity u^*) and g is gravity acceleration.

The wind speed scaling (8) in experimental parameterizations (6, 7) determines 'traditional vision' of wind-wave interaction: this scaling implies universality of the interaction and, as a consequence, universality of exponents and pre-exponents in (6, 7). All inconsistencies of the resulting dependencies (6,7) with this concept of universality of wave growth are treated as 'imperfectness' of experimental set-up when additional features such as gustiness of wind, groupiness of wave field etc. accompany wind wave growth.

The weakly turbulent split balance model (1,2) gives a new vision of wave growth universality as a link of wave energy and net wave input. This concept of leading nonlinear transfer is free of details of wave generation and dissipation and, thus, it can be more useful for experimental verification. In this paper we overview results of more than twenty experimental studies in order to show their conformance with the weakly turbulent nature of wind-wave growth. Theoretical background and discussion of features of the collection of experimental dependencies are presented in (Badulin *et al.*, 2007).

2. EXPERIMENTAL POWER-LAW DEPENDENCIES OF WIND-WAVE GROWTH

Over the years, a great number of field experiments have been undertaken to fit the evolution of total wave energy (wave variance) and peak or mean frequency with simple dependencies on non-dimensional fetch. In these experiments wind is assumed to blow perpendicularly to a straight coast line, the wave field is stationary and developing in one spatial direction only while conditions in along-shore direction remain homogeneous. Just the fetch-limited set-up of sea experiments is generally considered as a reference case that provides a physically correct idealization for the Kitaigorodskii (1962, 1983) self-similarity approach. Following this approach wind speed is used as a scale of 'an effective fetch' and, dependence on this effective fetch can be simulated by measuring the waves at a single point. Similarly, duration-limited measurements in spatially homogeneous sea at stationary wind operate with single-point data and 'an effective duration'. The duration-limited measurements are always questionable for analysis because of difficulty to determine a reference time when waves start to grow. In some cases such data are converted to fetch-limited data by a heureistic rather than by a mathematically correct way (e.g. Hwang & Wang, 2004) to relate them with 'true' fetch-limited data.

Basing on the idea of 'effective fetch' (Kitaigorodskii, 1962) results of measurements at different conditions are often combined to derive 'more statistically reliable' wave-growth dependencies. Moreover, in a number of studies, this idea is extrapolated to essentially different physics, when waves in laboratory tanks and at sea are competing on equal terms in combined data sets for describing the wave growth dependencies. Thus, not all of the dependencies are expected to conform to the weak turbulence law (5). First, the theory is not applicable to laboratory conditions. Another big issue is the traditional scaling by the wind which, the use of 'effective fetch' in available integral dependencies. Generally, we cannot un-scale the experimental data and remove the wind.

Recent analysis by Zakharov (2005) demonstrated that exponents of the fetch-limited growth dependencies follow remarkably well the self-similarity relationship for exponents p_χ, q_χ in (4)

$$p_\chi = \frac{10q_\chi - 1}{2} \tag{9}$$

for six fetch-limited experiments. It is not the case for the comprehensive set of experiments presented in this paper. Moreover, for two thirds of the cases selected by Zakharov (2005) a coincidence rather than a firm agreement takes place: methods of measurements and data analysis could have corrupted the 'true' wave growth dependencies in those records significantly.

At the first glance, the experimental data presented as power-law approximations (7) are 'ready-to-use' for verification of the theoretical law (5). First of all, the exponents p_χ, q_χ are provided in explicit form and the corresponding theoretical linkage of these exponents (9) can be checked trivially in the spirit of Zakharov (2005). Secondly, the total wave input $\mathrm{d}\varepsilon/\mathrm{d}t$ (the convective derivative) can be calculated analytically for (4). In non-dimensional variables with constant scales of energy, frequency and fetch, it gives

$$\alpha_{ss} = \left(\frac{2\tilde{\varepsilon}_0^2\tilde{\omega}_0^{10}}{p_\chi}\right)^{1/3}\chi^{z_\chi} \tag{10}$$

where exponent

$$z_\chi = \frac{2p_\chi - 10q_\chi + 1}{3} \tag{11}$$

is a detuning of exponents p_χ, q_χ relatively to the theoretical relationship (9). Thus, to have a time-independent estimate of self-similarity parameter α_{ss} we need to consider one of the exponents p_χ, q_χ as 'more reliable'. Further, we assume that it is p_χ.

In this paper, we analyze all available dependencies of total wave energy and wave frequency collected in fetch-limited experiments over the past 50 years or so. As mentioned above, these dependencies should go through a thorough revision before they can be used for comparisons.

All the dependencies are listed in four groups (see Tables 1, 2) and the corresponding results are presented in different panels in figures 1, 2. We tried to follow formal criteria when created these lists. Sometimes these criteria were difficult to relate with particular experimental cases. Thus, the proposed ranging of the experiments is, to some extent, arbitrary and breaks between groups are not so strict.

2.1 Group I. 'The cleanest' dependencies

The first list (group I in Tables 1, 2) presents the 'cleanest' (from the point of view of our theory) results. Within the first group, we shall refer to the Black Sea experiment (Efimov *et al.*, 1986; Babanin & Soloviev, 1998) as a reference one, mainly, because the raw data are available for re-analysis. All other series of the group are based on measurements in a number of points: the dependence on non-dimensional fetch was not simulated by variation of the wind speed. Walsh *et al.* (1989) used airborne wave measurements, while wind speed was estimated by interpolation of both airborne and land (in JFK airport) data. Kahma & Calkoen (1992) analyzed thoroughly the point of possible correlation of wind speed and the resulting effective fetch. An additional feature of analysis by Kahma & Calkoen (1992) is in thorough discriminating of cases of different stratifications of atmospheric boundary layer. The resulting parameters of wave growth (cases 1.3 and 1.4 in Table 1, fig. 1) differ significantly. At the same time, conformance of exponents p_χ, q_χ with (5) is perfect and difference in self-similarity parameters α_{ss} for these two cases is quite small.

	Experiment	$\tilde{\varepsilon}_0 \times 10^7$	p_χ	$\tilde{\omega}_0$	q_χ	z_χ
1.1	Babanin & Soloviev (1998), Black Sea	4.41	0.89	15.14	0.275	0.010
1.2	Walsh et al. (1989), US coast	1.86	1.0	14.45	0.29	0.033
1.3	Kahma & Calkoen (1992) unstable	5.4	0.94	14.2	0.28	0.027
1.4	Kahma & Calkoen (1992) stable	9.3	0.76	12.	0.24	0.040
2.1	**Dobson et al. (1989)**	**12.7**	**0.75**	**10.68**	**0.24**	**0.033**
2.2	Kahma & Pettersson (1994)	5.3	0.93	12.66	0.28	0.020
2.3	JONSWAP by Davidan (1980)	4.363	1.0	16.02	0.28	0.067
2.4	JONSWAP by Phillips (1977)	2.6	1.0	11.18	0.25	0.167
2.5	Kahma & Calkoen (1992) composite	5.2	0.9	13.7	0.27	0.033
2.6	**Donelan et al. (1985)**	**8.41**	**0.76**	**11.6**	**0.23**	**0.073**
2.7	CERC (1977) by Young (1999)	7.82	0.84	10.82	0.25	0.060
3.1	Wen *et al.* (1989)	18.9	0.7	10.4	0.233	0.023
3.2	Evans & Kibblewhite (1990) neutral	2.6	0.872	18.72	0.3	-0.085
3.3	Evans & Kibblewhite (1990) stable	5.9	0.786	16.27	0.28	-0.076
3.4	**Kahma (1981,1986) rapid growth**	**3.6**	**1.0**	**20**	**0.33**	**-0.100**
3.5	Kahma (1986) average growth	2.0	1.0	22	0.33	-0.100
3.6	**Donelan et al.(1992), St. Claire**	**1.7**	**1.0**	**22.62**	**0.33**	**-0.100**
3.7	Hwang & Wang (2004); Hwang (2006)	6.19	0.81	11.86	0.237	0.084
3.8	Ross (1978), Atlantic, stable	1.2	1.1	11.94	0.27	0.167
3.9	Liu & Ross (1980), Michigan, unstable	0.68	1.1	12.88	0.27	0.167
3.10	Liu & Ross (1980), our fit	77	0.52	2.36	0.08	0.413
3.11	Davidan (1996) for u^* scaling	794.0	1.0	9.160	0.34	-0.133
4.1	**JONSWAP Hasselmann *et al.* (1973)**	**1.6**	**1.0**	**21.99**	**0.33**	**-0.010**
4.2	**Mitsuyasu et al. (1971)**	**2.89**	**1.008**	**19.72**	**0.33**	**-0.095**

Table 1: Exponents and pre-exponents of wind-wave growth in fetch-limited experiments. Cases studied in Zakharov (2005) are given in bold.

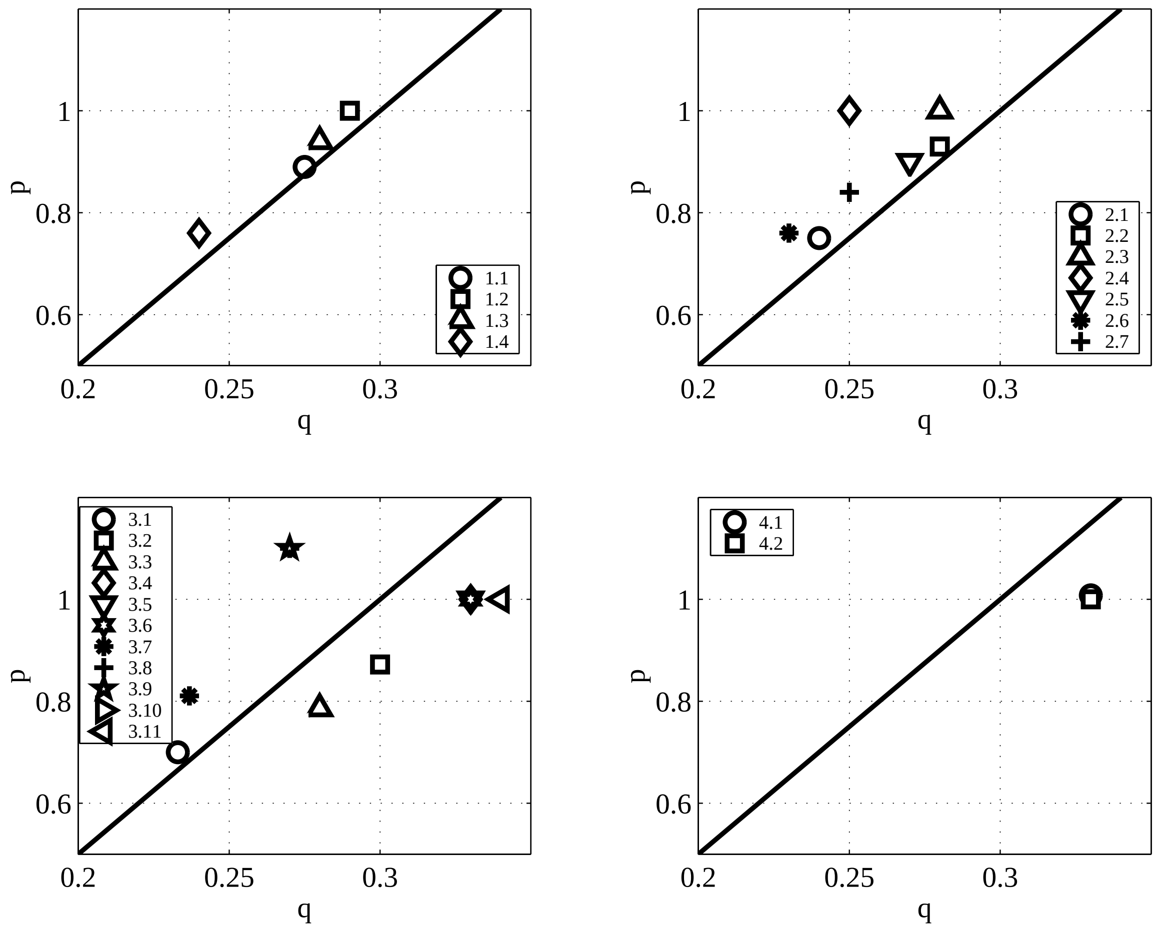

Figure 1: Dependence of p_χ on q_χ for different groups of fetch-limited experiments (sect. 4.2.1–4.2.4). Solid line — theoretical dependence $p_\chi(q_\chi)$ (9) for fetch-limited growth. Symbols for experiments collected in Table 1 are given in legends.

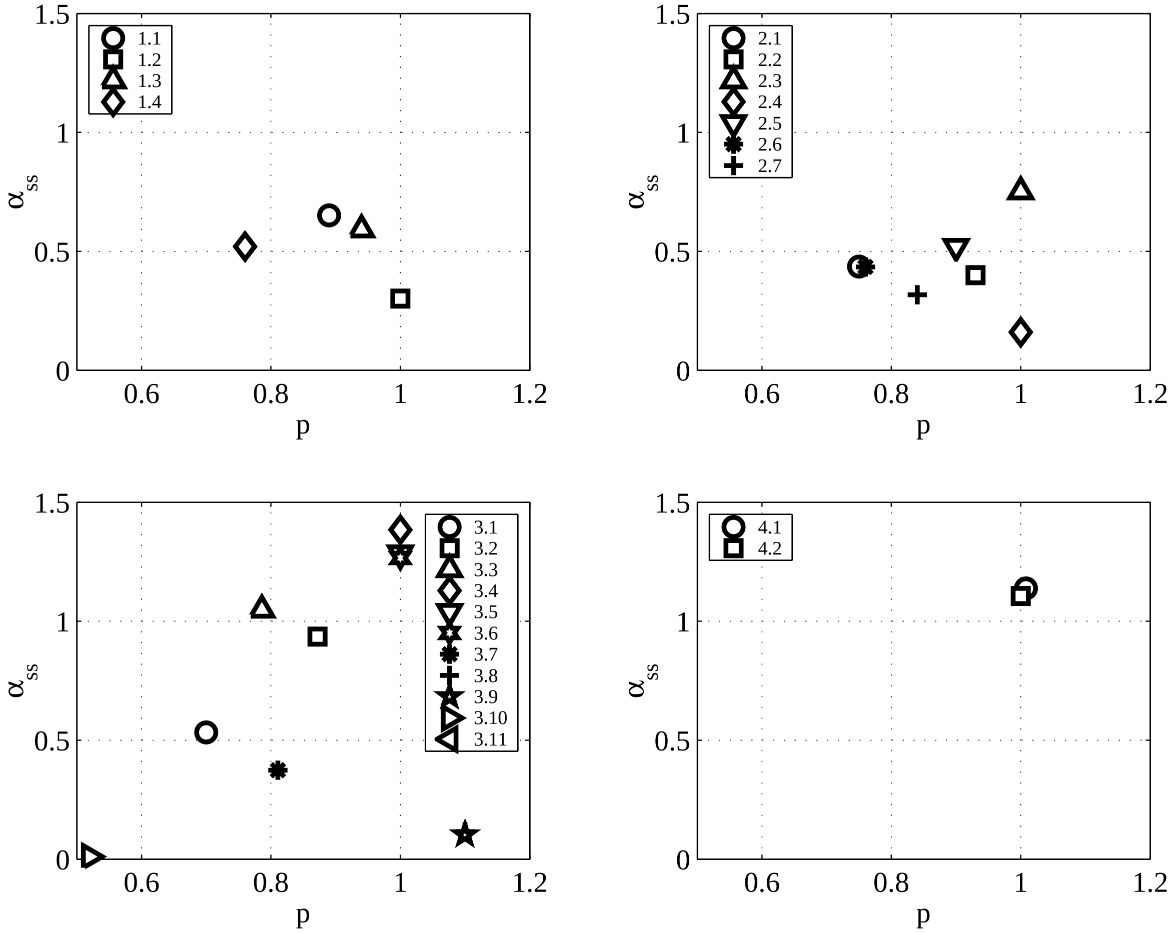

Figure 2: Dependence of α_{ss} on p_χ for fetch-limited experiments.

	Experiment	p_χ	z_τ	α_{ss}
1.1	Babanin & Soloviev (1998), Black Sea	0.89	0.010	0.652
1.2	Walsh et al. (1989), US coast	1.0	0.033	0.302
1.3	Kahma & Calkoen (1992) unstable	0.94	0.027	0.591
1.4	Kahma & Calkoen (1992) stable	0.76	0.040	0.520
2.1	**Dobson et al. (1989)**	**0.75**	**0.033**	**0.436**
2.2	Kahma & Pettersson (1994)	0.93	0.02	0.400
2.3	JONSWAP by Davidan (1980)	1.0	0.067	0.751
2.4	JONSWAP by Phillips (1977)	1.0	0.167	0.160
2.5	Kahma & Calkoen (1992) composite	0.90	0.033	0.519
2.6	**Donelan et al. (1985)**	**0.76**	**0.073**	**0.435**
2.7	CERC (1977) by Young (1999)	0.84	0.060	0.318
3.1	Wen *et al.* (1989)	0.7	0.023	0.533
3.2	Evans & Kibblewhite (1990), neutral	0.872	-0.085	0.936
3.3	Evans & Kibblewhite (1990), stable	0.786	-0.076	1.048
3.4	**Kahma (1981,1986) rapid growth**	**1.0**	**-0.100**	**1.385**
3.5	Kahma (1981) average growth	1.0	-0.100	1.286
3.6	**Donelan et al. (1992)**	**1.0**	**-0.100**	**1.266**
3.7	Hwang & Wang (2004,2006)	0.81	0.084	0.373
3.8	Ross (1978), Atlantic, stable	1.1	0.167	0.116
3.9	Liu & Ross (1980), Michigan, unstable	1.1	0.167	0.102
3.10	Liu & Ross (1980), our fit	0.52	0.413	0.011
3.11	Davidan (1996) for u^* scal.	1.0	0.340	3.743
4.1	**JONSWAP Hasselmann *et al.* (1973)**	**1.0**	**-0.100**	**1.106**
4.2	**Mitsuyasu et al. (1971)**	**1.008**	**-0.095**	**1.138**

Table 2: Exponent p_χ of wind-wave growth and self-similarity parameter α_{ss} in fetch-limited experiments for the observed p_χ and theoretical value of q_χ^{th}(9), z_τ is detuning exponent in formula for self-similarity parameter α_{ss} (10). Cases by Zakharov (2005) are given in bold.

2.2 Group II. Composite data

The series of the second list were obtained in fetch-limited experiments in a number of points and, in this sense, they are similar to those of the first list. At the same time, they may suffered some lack of accuracy in terms of our theory, first of all, due to composite data sets for different conditions of wave development following the idea of an 'ideal' set of wave growth exponents p_χ, q_χ and attempting to have 'statistically more reliable results'. Results of this group can be used for our analysis with some caution. They demonstrate a reasonably good conformance with the theoretical predictions. Spurious correlations of wind speed with effective fetch were avoided in some cases (e.g. cases 2.2,2.5) by thorough selection of wind wave data. As Kahma & Pettersson (1994) claimed (p. 262): 'the effective fetch concept is a poor approximation'.

For JONSWAP subsets (cases 2.3, 2.4) one has stronger deviation of the corresponding exponents (fig. 1) from the theoretical line and high dispersion of the estimates of self-similarity parameter α_{ss} (fig. 2). In terms of exponents p_χ, q_χ the subset by Davidan (1980) is in better agreement with theoretical dependence (9). At the same time, estimates of α_{ss} of both Davidan (1980) and Phillips (1977) differ significantly from all other estimates of groups I and II. A possible explanation of this outlier was proposed by Kahma & Calkoen (1992): 'many of the spectra from the JONSWAP experiment show more structure' compared to other experiments – i.e. their form is often not that due to generation by the wind only but reveals presence of mixed seas, swell etc.

An important feature of two groups of 'good' dependencies considered above is underestimating of experimental exponent q_χ as compared with theoretical dependence (9). This effect was explained in (Badulin *et al.*, 2007) as a result of spectra widening with wave growth.

2.3 Group III. 'Bad' dependencies

The third list is an antagonist of the first two groups: the collected dependencies were obtained for composite data sets and used one-point measurements with further conversion into dependencies on dimensionless fetch by varying the wind speed. These data are not expected to conform to our theory and therefore should not be used for comparison and verification of the dependencies for the self-similar wave growth. Also, parameterizations where the exponents were presumed on a basis of some grounds or considerations, and the dependencies were forced to fit these exponents, were placed into this group. Obviously, such presumed exponents may correspond to the theoretically expected exponents only by coincidence.

This group shows high dispersion of exponents of wave growth relatively to the theoretical dependence. Moreover, the corresponding points (fig. 1, left bottom) are scattered rather than to be above the theoretical line like for 'good' series. Estimates of self-similarity parameter α_{ss} show very strong variability as well. Note, that these estimates are affected strongly by experimental parameter $\tilde{\omega}_0$ which high power 10/3 in formula (10) can cause great errors of the estimates.

2.4 Group IV. Effect of wave tank data

The last group comprises of two, may be, the most respected experimental works on wave growth by Mitsuyasu *et al.* (1971) and Hasselmann *et al.* (1973). The formal reason for their low ranking is the use of laboratory measurements. Tank data are embedded in these experimental dependencies where they are combined with the field data. Laboratory waves, however, correspond to very short fetches (a few hundreds wave lengths at the best) and thus cannot be directly related to the open sea conditions as their physics is quite different. The kinetic description (the Hasselmann equation) is not applicable to such waves both because of the short time of wave development and due to the quasi-unidirectional propagation where the essentially two-dimensional four-wave resonances responsible for nonlinear transfer can be suppressed or modified.

Zakharov (2005) chose 'the most representative' dependencies for his study (given in bold in Tables 1, 2). Note, that only two of these six dependencies fall into the 'good lists'. As it was shown in this paper a thorough selection is required to relate correctly such dependencies to the proposed weakly turbulent law of wind-wave growth.

3. DISCUSSION AND CONCLUSIONS

'Traditional' vision of wind-wave growth is based essentially on the concept of universal wind speed scaling. This implies universality of wind-sea interaction, that, in its turn, suggests universality of atmospheric boundary layer which is a point of telling criticism last years. The present study refines and essentially extends the concept of universality of wind-wave growth making accent on inherent wave dynamics: nonlinear transfer appears to be a major physical mechanism that determines rigid link of wave spectra with net total wave input. This rigidity does not fix parameters of wave growth but gives a family of possible wave growth dependencies which parameters (exponents and pre-exponents in cases under study) are linked to each other. This flexibility of wave growth parameters in full agreement with theoretical predictions is reflected by essential variability of observed exponents of wave growth (e.g. Table 1).

The self-similarity parameter α_{ss} should be viewed, if compared with exponents $p_{\tau(\chi)}$, $q_{\tau(\chi)}$, as a more rigid feature of wave development: in spite of difficulties of estimating α_{ss} from experimental and numerical results (e.g. high powers of ω_p in eq. 10) for 'good series' it varies in a relatively narrow range which reflects the universality of energy-flux relationship. Experimental estimates of α_{ss} were obtained for the first time. At the moment we can recommend $\alpha_{ss} = 0.55 \pm 0.25$: more detailed estimates of this basic physical parameter is a subject of further studies.

Stress again the key result of our analysis: weakly turbulent nature of wind wave growth find its justification in integral characteristics of wave field — total wave energy and characteristic frequency.

The research was conducted under the US Army Corps of Engineers W912HZ-04-P-0172, ONR N00014-06-C-0130, INTAS-8014, Russian Foundation for Basic Research 07-05-00648-a, 07-05-92211, ofi-a-05-05-08027 and Russian Academy Program 'Mathematical methods of nonlinear dynamics'. This support is gratefully acknowledged.

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