

# A TWO-SCALE APPROXIMATION FOR NONLINEAR ENERGY TRANSFERS IN OBSERVED WAVE SPECTRA

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## 1. Introduction

As discussed in Resio and Perrie (2006), the basis of modern wave models is the nonlinear wave-wave interactions involving quadruplets. In most modern operational wave models such as WAM, quadruplet wave-wave interactions are simulated by the Discrete Interaction Approximation, commonly referred to as the DIA, as formulated by WAMD1 (1988). We have previously presented the Two-Scale Approximation (TSA), based on the separation of a spectrum into a broad-scale component and a local-scale (perturbation) component. TSA uses a parametric representation of the broad-scale spectral structure, while preserving the degrees of freedom essential to a detailed-balance source term formulation, by including the second order scale in the approximation. Previous tests have used idealized wave spectra, including JONSWAP spectra (Hasselmann et al., 1973) with selected peakednesses, finite depth tests, and perturbation cases. For these idealized cases, results suggest that the TSA can provide significantly increased accuracy in representations of nonlinear energy transfers compared to the DIA. To investigate the ability of the TSA to replicate nonlinear energy transfers in more general situations, comparisons will be made here with results from the full Boltzmann integral (FBI) for measured field spectra.

In this paper we present comparisons between the three formulations: TSA, DIA and FBI, using (a) observed wave spectra from Currituck Sound and (b) data from open ocean conditions from a directional waverider located off the US Army Field Research Facility at Duck, North Carolina during hurricane Wilma in 2005. The observed data from Currituck Sound were collected from a wave-staff array as described by Long and Resio (2007). The data are 2-d spectra in fetch-limited conditions, in onshore and offshore wind conditions and in slanting fetch conditions. Each Currituck Sound case represents a composite spectrum, consisting of data that were carefully analyzed from multiple measurements. On the other hand, open ocean waverider data collected during hurricane Wilma consist of individual open ocean directional wave spectral cases, as the storm moved northeastward from Florida to beyond Cape Hatteras, achieving maximum wave heights of 4.2 m.

Section 2 presents an overview description of the observed wave data from Currituck Sound and hurricane Wilma. Section 3 compares results from the three  $S_{nl}$  formulations using the wave spectra from Currituck Sound and from hurricane Wilma and Section 4 gives conclusions.

## 2. Observed Field Data

### a. Currituck Sound

The Currituck Sound wave data set consists of composite spectra grouped in five classes of inverse wave age ranging from 1.5 to 4.0 with bin widths of 0.5 and eighteen classes of wind direction using 20-deg bin widths, which collectively span a full circle (eighteen classes of 20-deg width, with two of the eighteen classes having too few observations to average). The highest relative frequency bins are  $ff_p = 3.5$ . Details are presented in Long and Resio (2007), illustrating 16 mean directional distributions, referenced to the Currituck Sound basin boundaries for each of the wind-direction classes, relative to mean wind direction. Resulting spectra are narrow directional distributions near the spectral peaks and broad distributions at higher wave numbers suggesting two modal peaks with the arc  $\Delta\theta$  separation of about 100 deg at higher wave numbers, similar to results of other researchers.

### b. Hurricane Wilma

Hurricane Wilma propagated on a storm track that was oriented to the northeast, passing over Florida and continuing to move along the coast. Although its minimum central sea level pressure exceeded category 5 storm intensity, it had weakened considerably by the time it passed by Cape Hatteras. Maximum winds recorded at the directional waverider (#630) near the Field Research Facility near Duck NC were  $14.6 \text{ ms}^{-1}$  and maximum significant waves were 4.13 m at 22 EST (Eastern Standard Time) on 24 October 2005.

We focus on the waves at the peak of the storm during the period from 07 EST on 24 October to 07 EST on 25 October, as wind-generated waves from Wilma grew to maximum values and then decayed as the storm moved past Cape Hatteras. During this time, the peak spectral wave direction was almost constant with an average direction of about  $45\text{-}65^\circ$ , although

wind directions changed continuously, ranging from 78.5° to 294.5°.

### 3. Comparisons of $S_{nl}$ formulations

All the Currituck Sound spectra cases were computed and compared. Figure 1 presents the 1 – d nonlinear transfer rates for one example, using the three  $S_{nl}$  formulations. Comparisons show that on the spectral forward face ( $f/f_p < 1.0$ ) DIA has some ability to compare with FBI results, whereas FBI and TSA results agree relatively well on the forward face. On the rear face of the spectrum ( $1 < f/f_p < 1.5$ ), DIA appears erratic and performs poorly relative to TSA, and TSA suggests a transfer rate that has a negative bias compared to FBI. In the equilibrium ranges ( $1.5 < f/f_p < 3.0$ ), DIA compares more closely in magnitude to FBI than does TSA, but has erratic tendencies, whereas TSA and FBI correlate well.

Overall, results suggest that TSA gives results that retain many of the FBI characteristics, with similar values and smooth behavior, whereas DIA seems unstable and tends to differ significantly from FBI.

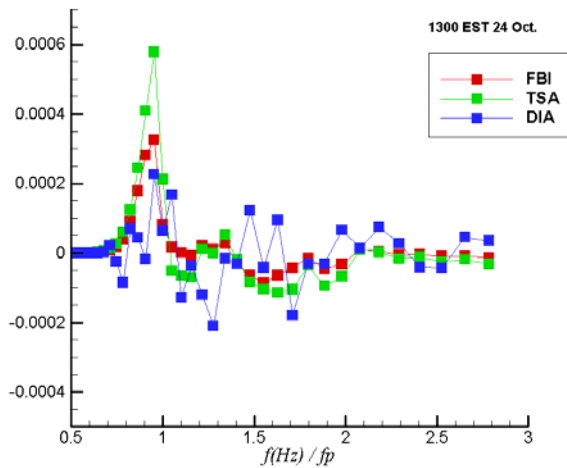


Figure 1. Currituck Sound spectrum located west (-110° to -130°) from the observation position.

Results from hurricane Wilma are suggested in Figure 2. This figure shows that TSA can largely simulate FBI behavior on the spectral forward face, rear face and the equilibrium range. By comparison DIA gives poor results, with only a rough qualitative approximation to the FBI behavior in all regions of the spectrum. Estimation of normalized root mean square errors (N RMSE) for all hurricane Wilma spectra during the period from 07 EST on October until 07 EST on 25 October are given in Figure 3, comparing TSA and DIA to FBI. As an overall result computed for the entire

spectrum, values for N RMSE computed for TSA are about half those resulting for DIA. Figure 3 suggests that over the spectral regions (forward face, rear face, and equilibrium range), N RMSE values for TSA are about one half of those for DIA. These results show no apparent variation with time.

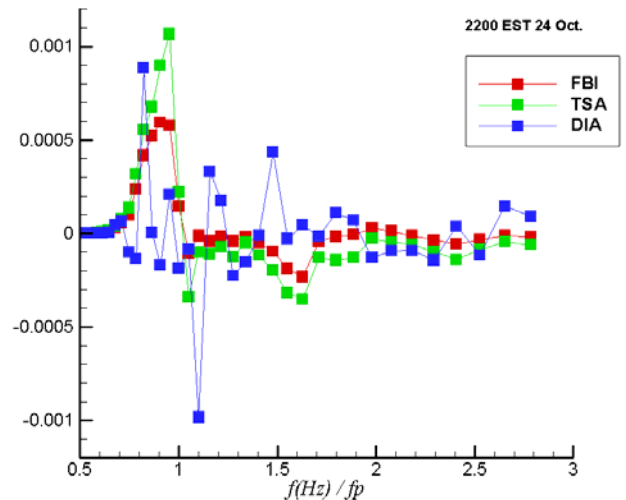


Figure 2. Normalized 1-d transfer with TSA and FBI compared with DIA, for the wave spectra observed by the directional waverider off Duck NC during hurricane Wilma (time and date indicated in panel), for developing wind-sea spectra.

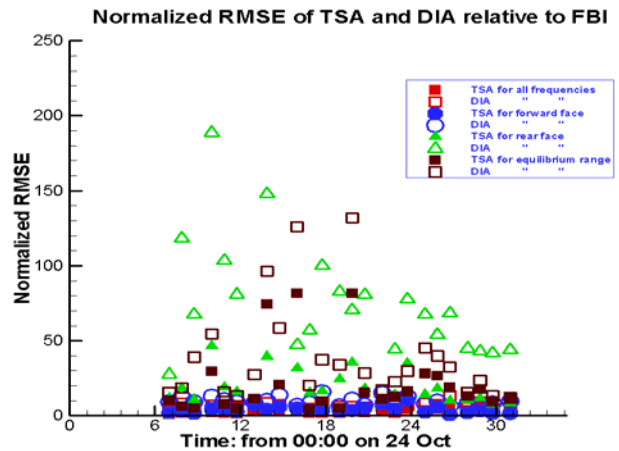


Figure 3. Error calculations for hurricane Wilma spectra cases showing normalized RMSE.

### 4. Conclusions

We have presented comparisons between the three formulations: TSA, DIA and FBI, using (a) observed wave spectra from Currituck Sound (Long and Resio, 2007) and (b) directional waverider data collected from

open ocean conditions off the US Army Field Research Facility at Duck, North Carolina during hurricane Wilma.

In general, it is important for the parametric terms in the TSA to capture much of the broad-scale behavior of the spectra. However, even for complicated cases in which observed spectra deviate from simple parametric forms, the perturbation component of the TSA does a relatively good job of forcing the total TSA solution toward the FBI solution. In comparisons from Currituck wave classes or hurricane Wilma data, TSA and FBI compare well, in 1-d and 2-d results (not shown), particularly in the region of the low-frequency side of the spectral peak. However, although DIA is somewhat comparable with FBI in 1-d comparisons for the forward face, rear face and equilibrium range, it is not competitive with TSA and correlates poorly with FBI. Overall, DIA exhibits less prominence of regions of positive transfer rates, compared to TSA or FBI. Qualitative DIA differences are significantly dissimilar.

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