Implementation of New Experimental Input/Dissipation Terms for Modelling Spectral Evolution of Wind Waves

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Abstract

Numerical simulations of the wind-wave spectrum are conducted on the basis of the new wind input and wave dissipation functions obtained in the Lake George field experiment. This experiment allowed simultaneous measurements of the source functions in a broad range of conditions, including extreme wind-wave circumstances.

Results of the experiment revealed new physical mechanisms in the processes of spectral input/dissipation of wave energy, which are presently not accounted for in wave forecast models. These results were parameterised as source terms in a form suitable for spectral wave models.

The simulations were conducted by means of the two-dimensional research WAVETIME model with an exact solution for the nonlinear term. Physical constraints were imposed on the source functions in terms of the known experimental dependences for the total wind-wave momentum flux. Enforcing the constraints in the course of wave spectrum evolution allowed fine tuning of experimental parameters of the new input and dissipation functions.

The resulting time-limited evolution of integral, spectral and directional wave properties, based on implementation of the new physically-justified source/sink terms and constraints, is then analysed. Good agreement of the simulated evolution with known experimental dependences is demonstrated.

1. Introduction

The dissipation term S_{ds} is one of the three most important source functions of the radiative transfer equation (RTE) employed by all spectral wave models to predict wave spectrum *F*:

$$\frac{dF}{dt} = S_{in} + S_{nl} + S_{ds} + ... \qquad (1)$$

where the two other sources of wind input S_{in} and resonant nonlinear four-wave interactions S_{nl} are also explicitly mentioned. In a general case, all the source terms as well as the spectrum itself, are functions of wavenumber ***k*,** frequency ω, time t and spatial coordinate ***x*.**

Since the major, if not dominant part of S_{ds}, is attributed to energy losses due to wave breaking, and breaking has been routinely regarded as a poorly understood and basically unknown phenomenon, formulations of the term have always been loosely based on physics and served as a residual tuning knob (e.g. Cavaleri et al., 2007,

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Babanin and van der Westhuysen, 2007). This paper is a companion paper to Babanin et al. (2007a) where it is shown that such an approach is no longer satisfactory. Over the past decade, many physical features of the dissipation performance were discovered experimentally and are now well understood and described. Among them, the threshold behaviour of wave breaking (Banner et al., 2000, Babanin et al., 2001, Banner et al. 2002), the cumulative effect of wave dissipation at smaller scales (Donelan, 2001, Babanin and Young, 2005, Young and Babanin, 2006), quasi-singular behaviour of dissipation in the middle wavelength range (Hwang and Wang, 2004), two-phase behaviour of the dissipation (Babanin and Young, 2005, Manasseh et al., 2006), and alteration of wave breaking/dissipation at strong wind forcing (Babanin and Young, 2005).

How is this physics, which is by no means tentative reasoning, but a definite field observation, included in S_{ds} terms? In WAVEWATCH, two-phase behaviour of the dissipations term is accommodated (although assumed physics of Tolman and Chalikov (1996) is different to that in the experiments by Babanin and Young (2005)). Van der Westhuysen et al. (2007), in their SWAN model simulations, incorporated threshold limitations and a wind-forcing dependence for the dissipation function. Overall, however, most of models are missing on the new physics.

The observed features, nonetheless, do need to be accommodated in modern dissipation terms, otherwise the models do not reflect the correct physics and do not describe the reality adequately. It is rapidly becoming obvious that without incorporating these features the models cannot properly forecast complex or non-standard circumstances. Ardhuin et al. (2007) showed one such situation: wave growth in the presence of swell and at slanting fetches. The most apparent non-standard circumstance, where failure of the standard-tuned terms is to be expected, and which are also of the utmost interest from practical points of view, are extreme wind-wave conditions. As mentioned above, the dissipation function is altered in such circumstances, and so is the wind input (e.g., among most recent publications on this issue, Donelan et al., 2006, Stiassnie et al., 2007, Babanin et al., 2007b). No amount of good tuning and statistical fitting, as opposed to employing correct physics, will be able to extrapolate source terms tuned to standard conditions into such extreme situations.

Incorporation of the new dissipation features, however, is more complex than a mere replacing of one dissipation term with another. For example, if a cumulative integral is added to the breaking dissipation term, then local-in-wavenumber-space balance can no longer be satisfied and reformulations and readjustments of the wind input function, and perhaps of the entire model, will also be required. It is no surprise, therefore, that no model has tried to incorporate the cumulative effect so far.

This paper is dedicated to such an attempt based on a two-dimensional research model, WAVETIME, developed by van Vledder (2002). The wind-input function is not the main topic of the present paper. Since, however, as mentioned above, dissipation with the cumulative term requires a major revision of the accompanying wind-input formulation, in Section 2 this revision will be outlined.

2. The approach

The traditional approach to testing source functions employed in RTE (1) was laid by Komen et al. (1984) and with some variations has persisted throughout more than 30 years. Its main emphasis rests on implementing a new function in RTE, solving this equation numerically by means of a spectral wave model, and comparing the outcomes with known experimental results. These outcomes present a time/space development of integral, spectral and directional properties of the wave field. The evolution of such wave properties has been extensively investigated and is well understood, described and parameterised, much better than physics of the source terms S_{in} and particularly S_{ds}. Therefore, attention of the traditional approach concentrates on reproducing the known wave growth curves and some spectral features (e.g. Banner and Young, 1994), and the ability of a model to replicate these curves serves as a validation measure for the source terms.

Such validation criterion, however, is potentially subject to serious defectiveness. The wind input term supplies influx of the energy/momentum into the wave system, whereas the dissipation term provides the outflux. Together, they can balance each other and produce correct growth curves while being physically inadequate individually. For instance, nothing stops S_{in} to be such that its integral is tens of times greater than a realistic total wind energy input, provided that S_{ds} has a matching integral.

An additional disadvantage of the traditional approach is due to the impossibility to investigate and verify the source terms separately. Any change to any of the source terms requires extensive testing with a full spectral model which is computationally very expensive, particularly as the exact nonlinear term has to be solved. Besides, alterations of one term often leads to compensative tuning of other source functions which smear the physics of the original update.

A new approach has been developed which overcomes the above-mentioned limitations. The approach is elaborated and detailed in Tsagareli (2007) and is based on physical constraints suggested by Mark Donelan in the discussion at WISE-2004 meeting in Reading, England.

The main constraint imposes the condition that the integral wind momentum input S_{in}^{m} must be equal to the total wave-induced stress τ_w (i.e. total wind stress less the viscous component):

$$\int_0^{f_\infty} S_{in}^{m}(f)df = \int_0^{f_\infty} \frac{k}{\omega} S_{in}(f)df = \tau_w . \tag{2}$$

In the present study, $f_\infty = 10\,Hz$ was chosen such that it is well within the capillary wave-frequency range. Based on this constraint, behaviour of a new wind-input function can be investigated and tuned separately from any other term since dependences of the total drag on, for example, wind speed (e.g. Garratt, 1977) or wave age (e.g. Guan and Xie, 2004) are well known, as well as experimental dependences for the viscous drag (e.g. Banner and Peirson, 1998). The new experimental wind-input function employed in this study is that by Donelan et al. (2006) obtained in the Lake George experiment.

The second main constraint relates to the dissipation function:

$$\int_0^{f_\infty} S_{ds}(f)df \le \int_0^{f_\infty} S_{in}(f)df . \tag{3}$$

The ratio of the two integrals as a function of wave development stage is also known experimentally (e.g. Donelan, 1998), and therefore the dissipation term can also be studied and tuned individually. This will be discussed in Section 3 below.

The ability of the updated source terms to reproduce the growth curves and known behaviour of the wave spectral evolution are still obviously crucial. In the present approach, however, these checks are done at the final stage as shown in Section 4.

3. The new dissipation function

The new dissipation function was suggested on the basis of a series of experimental studies (Babanin and Young, 2005, Young and Babanin, 2006, Manasseh et al., 2006). New physical features of the spectral dissipation, such as threshold behaviour and cumulative effect at smaller scales have been verified by independent and redundant means before being incorporated in the suggested dissipation function (see Babanin et al., 2007a):

$$S_{ds}(f) = -a_1 \rho_w g f ((F(f) - F_{thr}(f))A(f))^n - a_2 \rho_w g \int_{f_p}^{f} ((F(q) - F_{thr}(q))A(q))^n dq \,. \quad (4)$$

Here, ρ_w is the water density, g is the gravitational constant, $A(f)$ is the integral characteristic of the inverse directional spectral width (Babanin and Soloviev, 1998a):

$$A(f)^{-1} = \int_{-\pi}^{\pi} K(f,\varphi) d\varphi \,, \quad (5)$$

where φ is the wave direction, $K(f,\varphi)$ is the normalised directional spectrum:

$$K(f,\varphi_{\max}) = 1 \,. \quad (6)$$

a_i are experimental constants, and $F_{thr}(f)$ is the spectral threshold function.

The dissipation function S_{ds} (4) accommodates the two-phase behaviour: it is a simple function of the wave spectrum at the spectral peak (inherent breaking) and has an additional cumulative term at all frequencies above the peak (induced dissipation). The induced dissipation at a particular frequency above the peak can be caused by lower-frequency breaking (i.e. Young and Babanin, 2006, Manasseh et al., 2006) or/and by modulation of the short-wave steepness by underlying longer waves (Donelan, 2001).

Formulation (4) has a number of parameters which require experimental measurements or numerical simulations to be defined: exponent n, spectral threshold $F_{thr}(f)$, and coefficients a_i. In this study, linear dissipation $n=1$ was employed which is consistent dimensionally, agrees with measurements by Young and Babanin (2006) and is best suitable for satisfying the physical constraints in the numerical simulations (i.e. coefficients a_i have to stay positive at all times).

Limits for the threshold value $F_{thr}(f)$ were identified in the experimental study of Babanin and Young (2005). They investigated this threshold in dimensionless terms, i.e. in terms of the saturation spectrum $\sigma(f)$ normalised by the directional spectrum parameter (5):

$$\sigma(f) = \sigma_{Phillips}(f) A(f) \quad (7)$$

where $\sigma_{Phillips}(f)$ is as introduced by Phillips (1984):

$$\sigma_{Phillips}(f) = \frac{(2\pi)^4 f^5 F(f)}{2g^2}. \quad (8)$$

If a universal dimensionless saturation-threshold value σ_{thr} can be established, the dimensional threshold can then be obtained at every frequency:

$$F_{thr} = \frac{2g^2}{(2\pi)^4} \frac{\sigma_{thr}}{A(f) f^5}. \quad (9)$$

According to Fig.7 of the companion paper by Babanin et al. (2007a), the bulk of Lake George data corresponds to values of the dimensionless threshold in the range of

$$\sqrt{\sigma(f)} > 0.035. \quad (10)$$

With this limit in mind, simulations were run for a broad range of the 10m-height wind speeds of $U_{10} = 7 - 20\, m/s$, and an additional restriction was imposed that at the full development (Pierson-Moscowitz stage) no breaking can occur at the spectral peak. This was found to be the case if the threshold chosen is:

$$\sqrt{\sigma_{thr}(f)} = const = 0.035. \quad (11)$$

Further extensive testing was required to identify values and behaviour of the coefficients a_1 and a_2. For the single record analysed, Young and Babanin (2006) obtained a_1=0.0065 and assumed the same value for a_2, but obviously in a general case this issue had to be revisited.

As mentioned in Section 2 above, the major constraint for the dissipation function (4) is the condition (3). A ratio of the dissipation integral to the input integral was adopted from the experiment of Donelan (1998). This ratio stays within the range of 95-100% for most of the wave development, reaching 100% at the Pierson-Moscowitz limit. It is only at very early stages that the total wind input is significantly larger than the total dissipation.

To estimate coefficient a_1, a balance of the wind input and dissipation below the spectral peak was used in a broad range of wave development stages of $U_{10}/c_p = 0.8 - 5.7$ where c_p is phase speed of the spectral-peak waves. According to the two-phase behaviour of the spectral dissipation (Babanin and Young, 2005, Manasseh et al., 2006), the induced-dissipation term of formulation (4) can be neglected below the peak and thus the entire dissipation in this region was attributed to the inherent breaking.

For the numerical experiments, a special JONWAP-like spectral form was adopted which had subintervals of f^{-4} and f^{-5} behaviour in the equilibrium interval. The first subinterval, closer to the spectral peak, is consistent with both observations (e.g. Donelan et al., 1985) and theory (e.g. Pushkarev et al., 2003). If, however, extended to $f_\infty = 10\, Hz$, it was found that under no circumstances could the spectrum satisfy the principal constraint (2). Thus, the Phillips (1958) f^{-5} equilibrium interval had to be re-introduced at higher frequencies. Due to the cumulative breaking behaviour, a lot of wave breaking is predicted at these frequencies and thus the original Phillips' idea can be applied. According to this idea, if the wave breaking dominates the dynamics and thus defines the spectral shape at certain scales, f^{-5} behaviour of the spectrum should be expected. The transition from f^{-4} to f^{-5} spectrum was determined from investigating the wind input spectral function (Tsagareli, 2007).

Once a_1 was obtained this way, balance between the total dissipation and total wind input for the variety of wave ages was considered in the whole spectral range and coefficient a_2 was also determined. Dependence of the two coefficients on the wave age U_{10}/c_p is shown in Fig.1. The straight line in this figure signifies the single value of a_1=0.0065 of Young and Babanin (2006) obtained at $U_{10}/c_p \approx 5$, and is consistent with the numerically inferred values for this coefficient at stronger wind forcing. Kinks on the simulated lines for a_1 and a_2 are due to the wind input going through the stage of the full flow separation (Donelan et al., 2006).

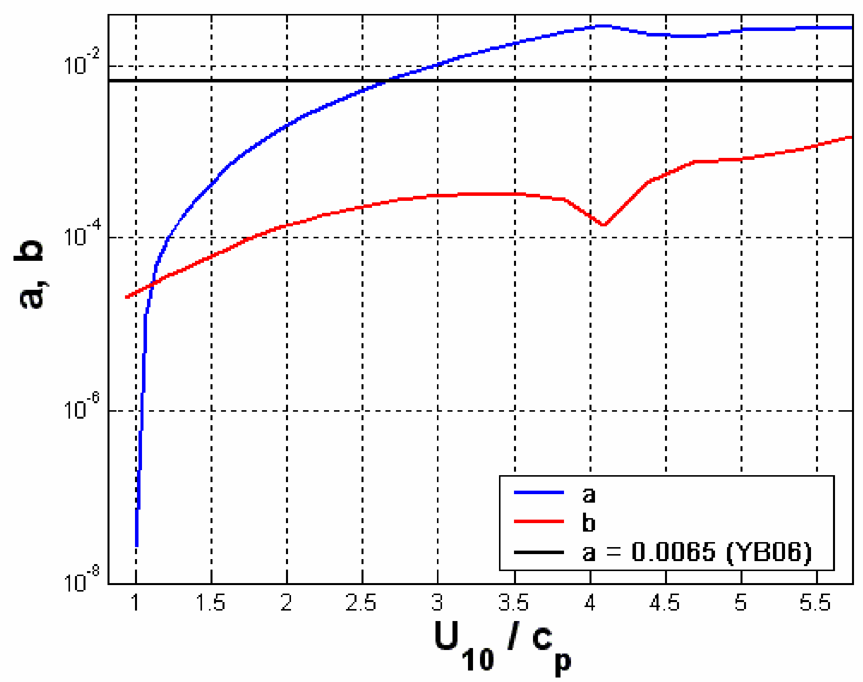

Figure 1. Dependence of coefficients a_1=a (top, blue line) and a_2=b (bottom, red line) on the wave development stage U_{10}/c_p. a_1=0.0065 of Young and Babanin (2006) is shown with straight line.

4. Modelling the wave evolution

Once the experimental properties of the dissipation function (4) were defined, numerical simulation of the wave evolution was conducted by means of WAVETIME model with exact computation of the nonlinear integral (van Vledder, 2002). In Fig.2, integral dependence of the total wave energy ε on the sea state is shown. The red dashed line signifies the experimental dependence of Babanin and Soloviev (1998b) with the 95% confidence intervals (dotted line). The agreement is good throughout the entire wave development.

Most difficult of all the wave spectral properties to simulate is the directional-spread behaviour (e.g. Banner and Young, 1994). The newly observed experimental feature of bimodality of the directional distribution of the dissipation function (Young and Babanin, 2006, see also the companion paper Babanin et al., 2007a) provides an additional flexibility in this regard. Along with the uni-directional dissipation function, such bimodal spread was also employed in the simulations (Fig.3).

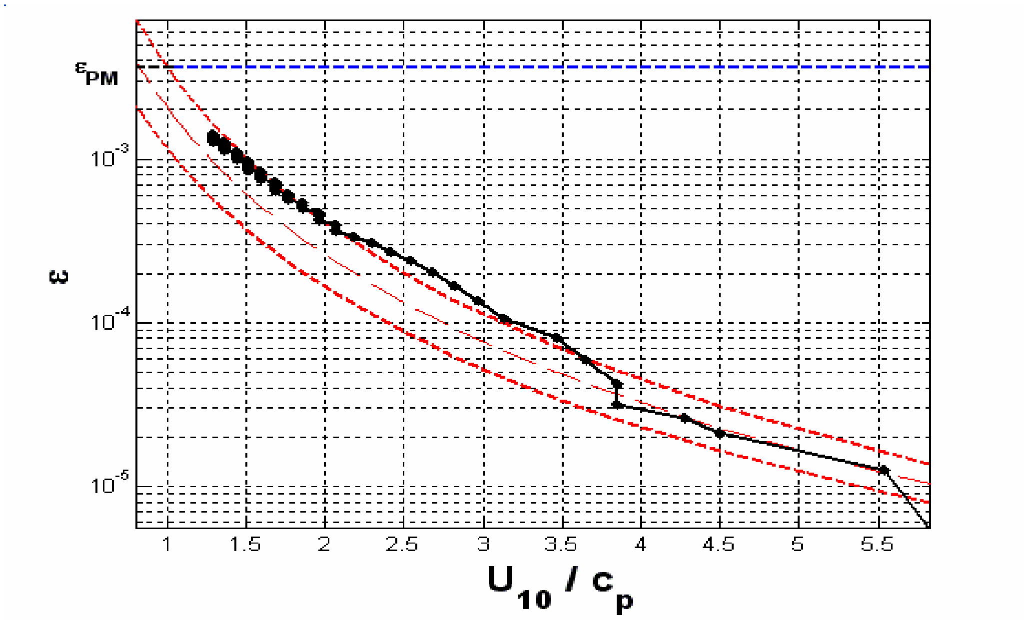

Figure 2. Dependence of the total wave energy ε *on the sea state* U_{10}/c_p*. The red dashed line signifies the experimental dependence of Babanin and Soloviev (1998b) with the 95% confidence intervals (dotted line). The blue dashed straight line is the Pierson-Moscowitz limit.*

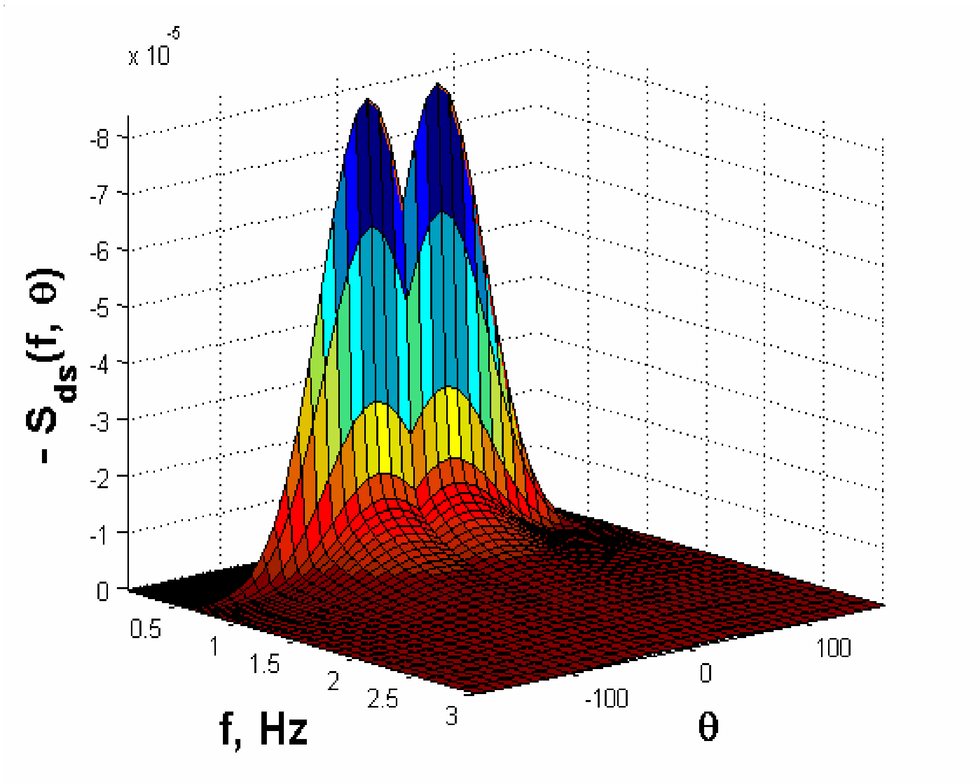

Figure 3. Example of a directionally-bimodal dissipation function S_{ds} .

In Fig.4, variation of the directional-width property *A(f)* (Eq.5) is shown obtained with the use of the bimodal distribution for the dissipation function, and compared with the experimental dependences of Babanin and Soloviev (1998a). Behaviour of the angular spread at the spectral peak is reproduced very well. For the higher frequencies and younger waves, the agreement is also good, but for later wave-development stages some limitations are still to be addressed.

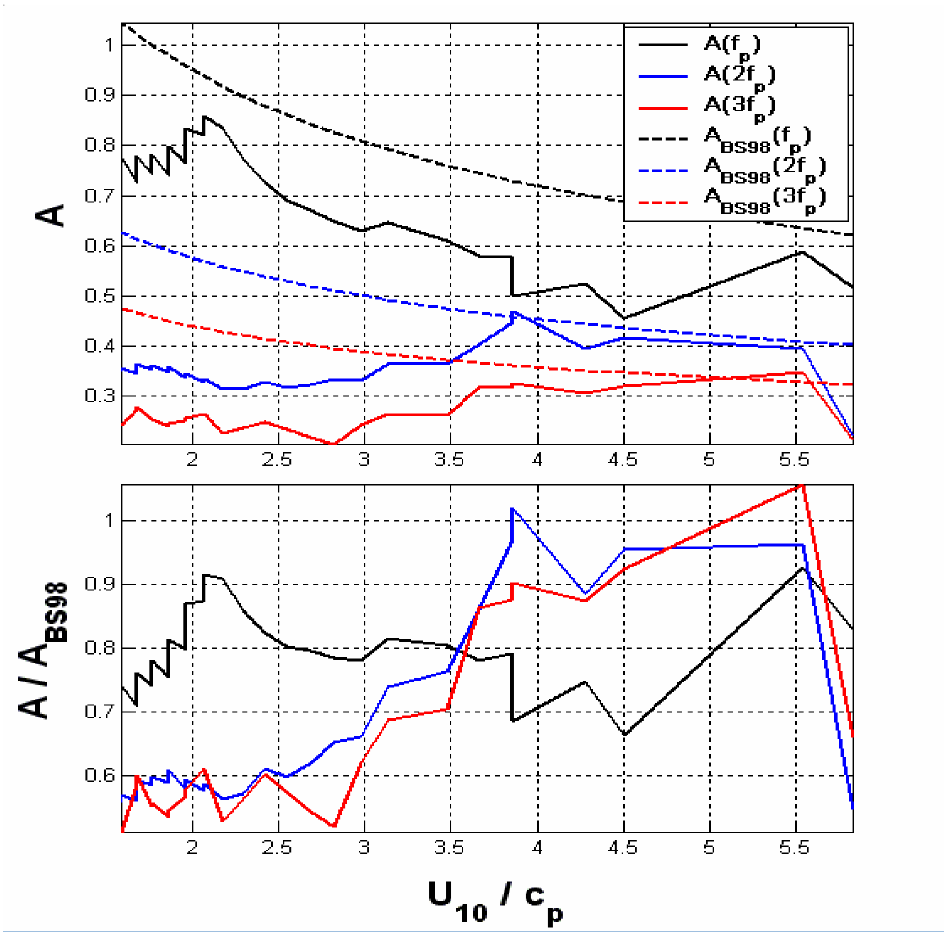

Figure 4. (top) Directional-width parameter A(f) (Eq.5) versus sea state U_{10}/c_p at different frequencies with respect to peak frequency f_p. Solid lines correspond to the numerical simulations, dashed lines – to the respective experimental dependences by Babanin and Soloviev (1998a). (bottom) Ratio of the simulated and experimental dependences for the three relative frequencies.

5. Discussion and summary

Many further comparisons and verifications for the new dissipation function are still to be conducted, but it is most interesting to follow the logic of the companion experimental paper by Babanin et al. (2007a, their Fig.10) and compare values of the coefficient *b* produced in these simulations and in the experiments (see definition of *b* in Eq.(13) below).

One of the purely white-capping properties, which can be easily converted into spectral dissipation, is *Λ(c),* the average length of breaking crests per unit area per unit interval of phase speed *c* (Phillips et al., 2001). Melville and Matusov (2002) obtained the spectral distribution of *Λ(c)* experimentally as a function of *c:*

$$\Lambda(c)\left(\frac{10}{U_{10}}\right)^3 = 3.3\times10^{-4}e^{-0.64c} \qquad (12)$$

which can then be converted into a dissipation function

$$S_{ds}(c) = b\rho_w g^{-1} c^5 \Lambda(c)(\frac{10}{U_{10}})^3 \quad (13)$$

and

$$S_{ds}(f) = \frac{g}{2\pi}\frac{1}{f^2} S_{ds}(c). \quad (14)$$

Here, b is an empirical parameter which has been shown to vary in a very broad range, almost by four orders of magnitude (Melville and Drazen, presentation at WISE-2007, Lorne, Australia). Remarkably, this is a similar range of the variation which is produced by the numerical simulations utilising the new dissipation function (Fig.5).

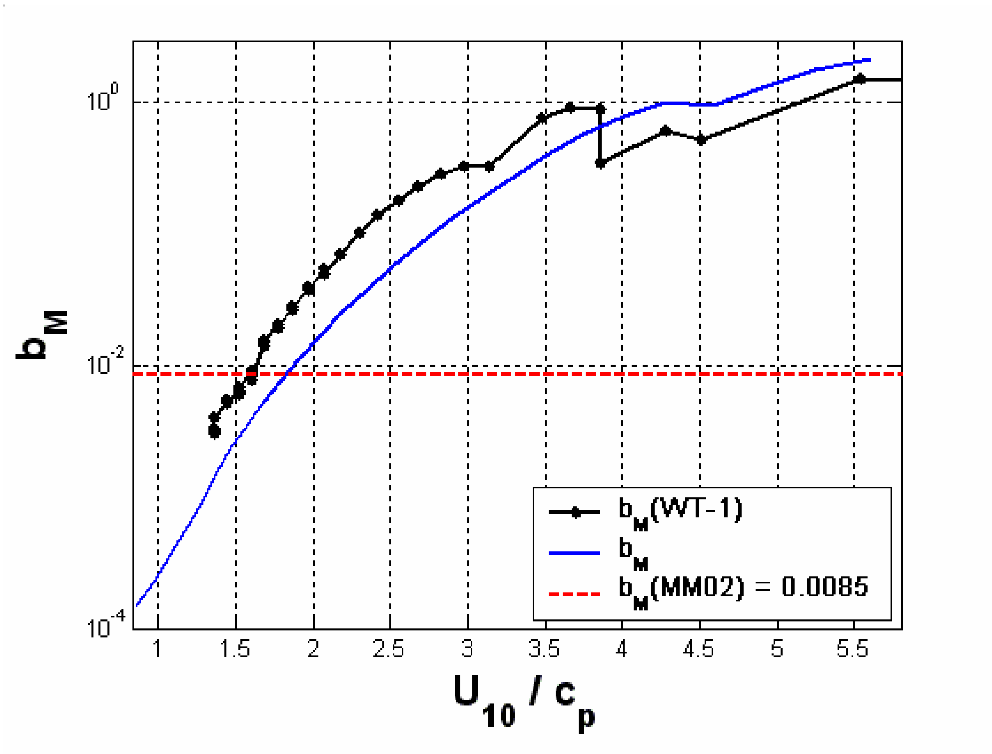

Figure 5. Dependence of the parameter b *(Eq.13) on the sea state U_{10} / c_p. The black line with dots is the result of the numerical simulations. The red dotted line is the value obtained in Melville and Matusov (2002).*

Finally, we would like to summarise the results of the present paper. A new dissipation function S_{ds} (Eq.4) has been suggested on the basis of experimental results (see companion paper Babanin et al., 2007a). It contains new physical features, such as threshold behaviour, cumulative effect at smaller wave scales, and bimodal directional distributions. These features, while are definite properties of the wave dynamics, are not presently employed by any other dissipation term used for wave spectral modelling at present.

An attempt to incorporate this term in a research spectral model is described. A new approach was employed which is based on strict physical constraints both for wind energy input and wave energy dissipation: i.e. the integral of the wind input must agree with experimentally observed values for the total stress, and the integral of the wave energy dissipation must satisfy experimentally measured ratios of the total input and total dissipation. Such an approach also allows the investigation of the source terms separately, before employing them in a wave model.

The approach allowed the determination of characteristics of the dissipation function (4) which have not been obtained experimentally, and the function was subsequently applied, along with the Donelan et al. (2006) wind-input term, to reproduce the wave evolution. Given the fact that no extensive tuning of new source functions in the course of the evolution runs were done, and all their properties were kept consistent with the experimental physics observed, the agreement of the numerical simulations with the experiment is very good. Although a significant effort is still to conducted before the new source functions can be employed in operational forecast models, both the new approach and the new source functions appear to be promising.

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